Non-Perturbative calculations in (Intense-Field) QED

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Outline

Why Study QED in Intense Fields??

QED has been extensively studied and our understanding of **perturbative** QED is extremely good,

$\Rightarrow$ e.g. anomalous magnetic moment of electron: theory and experiment have 10 decimal place agreement.

However, when considering QED in intense EM fields, we enter a non-perturbative regime, which is by comparison, poorly understood.

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However, when considering QED in intense EM fields, we enter a non-perturbative regime, which is by comparison, poorly understood.

$$\text{"Ultra-strong" fields:} \quad E_c = \frac{m^2 c^3}{e\hbar} \simeq 10^{16}\ V/cm$$

$$\text{"Intense" fields:} \quad I_c = \frac{c}{8\pi} E_c^2 \simeq 4 \times 10^{29}\ W/cm^2$$

So What?

- Breakthroughs in laser field intensities are slowly allowing experiments to probe a previously unexplored region of QED parameter space:

  Current records: $E \sim 10^{12}\ V/cm \qquad I \sim 10^{22}\ W/cm^2$

- Require better understanding of beam-beam interactions at beam crossings at linear collider, where field strengths also very high.

  $\rightsquigarrow$ theory currently behind experiment

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- Medical Applications: tumour therapy, medical imaging.

Throughout this talk:

- our approach is **semi-classical**, i.e. we treat the external field as unquantized
- consider only **plane-wave** external fields,

$$\text{i.e.} \quad A_\mu^{\text{ext}} = A_\mu^{\text{ext}}(k \cdot x)$$

Why the Classical Approximation?

Basic things we want to compute are **scattering amplitudes and probabilities**.

So need a way of descibing the external field as a special in/out photon state.

$\longrightarrow$ For intense fields, way to do this is via **coherent states**.

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So need a way of descibing the external field as a special in/out photon state.

$\rightarrow$ For intense fields, way to do this is via **coherent states**.

These are eigenstates of the positive frequency part of the photon field **operator** $\hat{A}_\mu$:

$$\hat{A}^{+}_{\mu}(x)\,|\,a\,\rangle = |\,a\,\rangle a_\mu(x)$$

and, each coherent state corresponds to a classical solution of the wave equation.

[Theory of coherent states developed by R. J. Glauber in 60's]

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Ok, so how does using these coherent states help??

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$$\langle\, p_i, \ldots, l_j,\; a \,|\, \hat{S} \,|\, p_k^{'}, \ldots, l_n^{'},\; a \,\rangle = \langle\, p_i, \ldots, l_j \,|\, \tilde{S}(a) \,|\, p_k^{'}, \ldots, l_n^{'} \,\rangle$$

$$\Downarrow$$

the change in the S-matrix from $\hat{S} \to \tilde{S}$ is realised through a change in our interaction lagrangian.

$$\mathscr{L}^{\text{int}}_{\text{photon}} \to \mathscr{L}^{\text{int}}_{\text{photon}} + \mathscr{L}^{\text{int}}_{\text{classical}}$$

where

$$\mathscr{L}^{\text{int}}_{\text{photon}} = -e\bar{\psi}\gamma^{\mu}\hat{A}_{\mu}\psi$$
$$\mathscr{L}^{\text{int}}_{\text{classical}} = -e\bar{\psi}\gamma^{\mu}A^{\text{ext}}_{\mu}(x)\psi$$

and $A^{\text{ext}}_{\mu}(x)$ now is just some c-function, not an operator.

...and why plane-wave backgrounds?

Taking a plane-wave background, $A_\mu^{\rm ext} = A_\mu^{\rm ext}(k\cdot x)$ is an idealisation but...

- Good representation of fields provided by laser beams.
- At Linear Collider it is good approximation to the field 'felt' by particles in one bunch due to field of oncoming bunch.
- In principle, any external field can be expanded in terms of plane-wave fields.
- Can make analytic progress with plane-wave external fields!

Basic Ingredients so far

Lagrangian we work with:

$$\mathscr{L} = \bar{\psi}(i\not{\partial} - m)\psi - e\bar{\psi}\gamma^\mu A^{\text{ext}}_\mu(x)\psi - e\bar{\psi}\gamma^\mu A_\mu\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$
$$= \mathscr{L}_{\text{QED}} - e\bar{\psi}\gamma^\mu A^{\text{ext}}_\mu(x)\psi$$

Fields we work with:

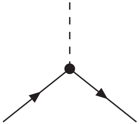
$$A^{\text{ext}}_\mu = A^{\text{ext}}_\mu(k \cdot x)$$

and are **classical**.

Traditional approach - Perturbation theory

"New Interaction" $\dashrightarrow$ new Feynman rule

$$-e\bar{\psi}\not{A}^{\text{ext}}\psi \quad \dashrightarrow$$



$$\sim (-ie)\int ds\not{A}^{\text{ext}}(s)\delta^{(4)}(p+sk-p')$$

So, can just do normal perturbation theory...right?

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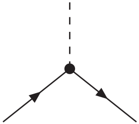

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For any process must do an all-orders series expansion in A^{ext}_μ.

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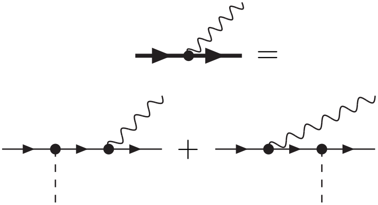

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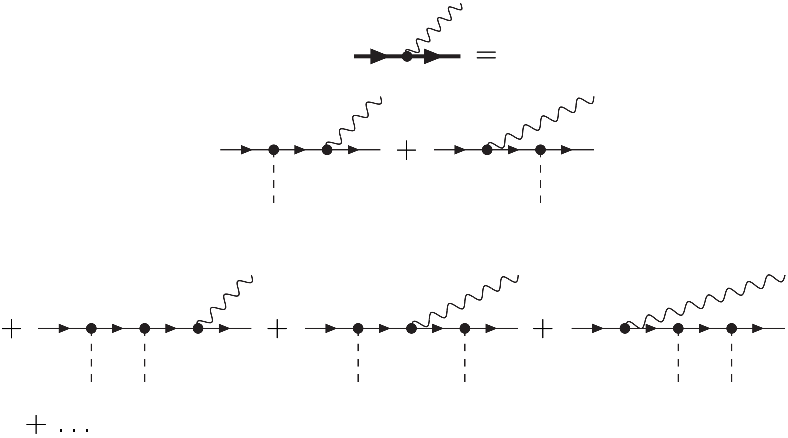

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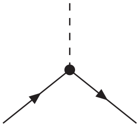

$$\sim (-ie)\int ds\,\not{A}^{\text{ext}}(s)\,\delta^{(4)}(p+sk-p')$$

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Wrong, have to remember, A_μ^{ext} is an intense field $\rightsquigarrow$ fixed order perturbation theory **no longer valid**.

For any process must do an all-orders series expansion in A_μ^{ext}.

$\longrightarrow$ **painful**

Volkov Functions (non-perturbative artillery)

Want to include the effects of the external field in an exact, non-perturbative manner.

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Idea:

$\longrightarrow$ include external field interaction as part of the free Hamiltonian

$\longrightarrow$ solve to find a new set of complete eigenstates (now encapsulating all effects of external field)

$\longrightarrow$ expand fields in terms of new eigenstates

$\longrightarrow$ only thing treat perturbatively is the ordinary photon field

How do we do this??

Volkov Functions (non-perturbative artillery)

Solve Dirac equation in external field:

$$\boxed{(i\not{\partial} - m - e\gamma^\mu A_\mu^{\text{ext}}(x))\psi = 0}$$

In general, this is difficult, but for plane-wave external fields we can solve this!

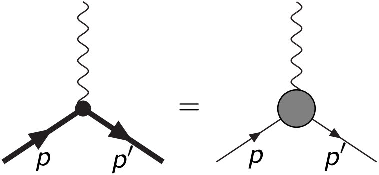
Conditions: $A_\mu^{\text{ext}} = A_\mu^{\text{ext}}(k \cdot x)$, $k^2 = 0$, $k \cdot A^{\text{ext}} = 0$

Solution:

$$\begin{aligned}\psi(x) &= E_p(x)u(p) \\ &= e^{-ip\cdot x}\left[1 + e\frac{\not{k}\not{A}}{2k \cdot p}\right] e^{-i\int^{k\cdot x}\left(e\frac{p\cdot A}{k\cdot p} - e^2\frac{A\cdot A}{2k\cdot p}\right)d\phi} u(p)\end{aligned}$$

[D. M. Volkov, 1935]

Result of all this is a new feynman rule (where all effects of external field are included **exactly**):



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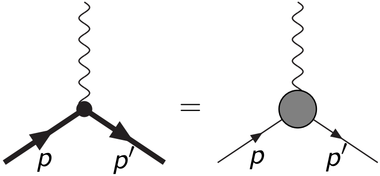


$\sim (-ie) \int d^4x \; \bar{E}_{p'}(x) \gamma^{\mu} E_p(x)$

$\rightarrow$ now proceed as normal and do perturbation theory with respect to the photon, using this new vertex.

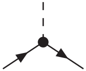
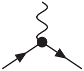
Does this really work...?

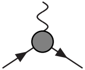
Do these Volkov functions really represent an ‘all-orders’ series expansion in A_μ^{ext}?

In fact, there is a unitary transformation relating two theories:

$$\mathscr{L}_1 = \bar{\psi}(i\partial\!\!\!/ - m - eA\!\!\!/^{\text{ext}} - eA\!\!\!/)\psi \;\cong\; \mathscr{L}_2 = \bar{\chi}(i\partial\!\!\!/ - m - eUA\!\!\!/U^{-1})\chi$$

[R. W. Brown & K. L. Kowalksi ,J. Kupersztych]

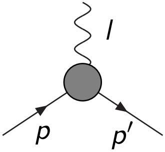
So, the theory with vertices/rules:  , 

is unitarily equivalent to the theory with vertex/rules: 

and actually $U = E_p$.

$\Rightarrow$ probabilities calculated using two methods are **equal**.

Simplest Example: Photon Emission



- Doesn't take place in absence of external field.
- Structure of tree level amplitude:

$$i\mathscr{A}^{\text{tree}} = (-ie)\int d^4x\, \bar{u}(p')\bar{E}_{p'}(x)\gamma^\mu E_p(x)u(p)\epsilon^*_\mu(l)e^{il\cdot x}$$

$$= \int_0^\infty ds\, \bar{u}(p')M^\mu(s,\ldots)u(p)\epsilon^*_\mu(l)\,\delta^{(4)}(p+sk-p'-l)$$

Circularly Polarized Field

$$A_\mu^{\text{ext}}(k\cdot x) = a_{1\mu}\cos(k\cdot x) + a_{2\mu}\sin(k\cdot x)$$

$$\text{with} \;\; a_1^2 = a_2^2 = -a^2, \;\; a_1\cdot a_2 = 0 = k\cdot a_1 = k\cdot a_2$$

$$\Downarrow$$

$$i\mathscr{A}^{\text{tree}} \to \sum_{n\geqslant 1} \bar{u}(p')\, M^\mu(n,\ldots)\, u(p)\, \epsilon_\mu^*(l)\, \delta^{(4)}(q + nk - q' - l)$$

[A. I. Nikishov, V. I. Ritus, ‘63,‘64]

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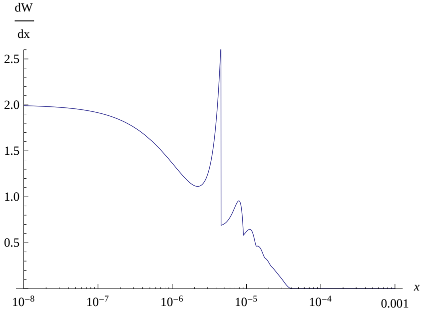
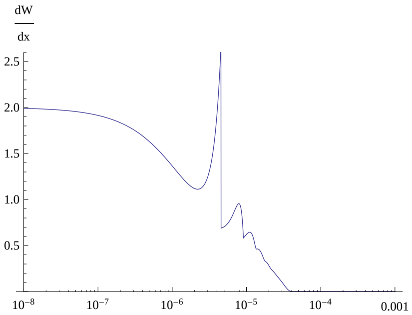
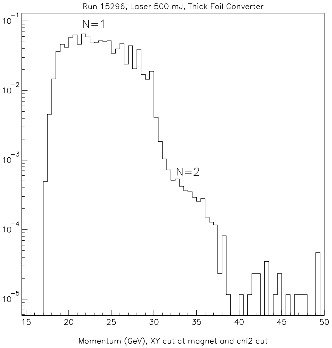
- $q^\mu = p^\mu - \frac{e^2a^2}{2k\cdot p}k^\mu$ is the so called 'quasi-momentum'

  $\Rightarrow$ "mass-shift" $m^2 \to m_*^2 = m^2(1-\frac{e^2a^2}{m^2})$ so far an unobserved effect...


dW/dx
2.5
2.0
1.5
1.0
0.5
x
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
0.001





[SLAC E-144 experiment]

Precision, precision...

We want to obtain as accurate and precise theoretical predictions as possible.

$\rightarrow$ to do this need to really understand any **singularities** present in amplitudes / cross-sections.

In normal QED: 2 types of singularities

- IR singularities from
  - real emission of soft photons
  - virtual corrections, loop momentum $\rightarrow 0$
- UV singularities from
  - loops, loop momentum $\rightarrow \infty$

What happens in External Field Case?

- Expect UV behaviour to still be there (need to renormalize)
- Don't yet know if/how external field affects IR singularities

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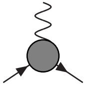
Step 1: look at $l \to 0$ behaviour of



Numerical investigation into complete decay for circularly polarized and constant crossed fields ($A_\mu^{\text{ext}} = (k \cdot x) a_\mu$) has revealed **no singularity**.

Don't yet know why it's not there...!

next on agenda to dig out the apparent non-existence of this singularity

Conclusions and Future Work

Conclusions

- Intense-field QED a good thing to study
- Volkov solutions as a non-perturbative, semi-classical approach
- New 'intense-field' effects

Outlook

- Look at processes with more vertices...more complicated calculations, but should shed light onto what precisely happens to IR-singularities
- Look at loops, how these corrections affect shapes of plots, renormalisation...

Thanks for listening!