Bayesian inference for hidden Markov models

Hidden Markov models, often abbreviated to HMMs, are models for time-series data that exhibit dependence over time. More specifically, a hidden Markov model is a bivariate discrete–time stochastic process $\{(\mathbf{Y}_t, S_t) : t = 1, \ldots, T\}$ consisting of an observed process $\{\mathbf{Y}_t : t = 1, \ldots, T\}$ and a hidden process $\{S_t : t = 1, \ldots, T\}$. The latter is often termed the *state* or *regime* and is usually assumed to have a discrete and finite state space. The former can be discrete or continuous, univariate or multivariate. In the standard theory of hidden Markov models this unobserved process is a first order Markov chain and, conditional on these hidden states, the observable random quantities, $\{\mathbf{Y}_t : t = 1, \ldots, T\}$, form a conditionally independent sequence in which the conditional distribution of $\mathbf{Y}_t$ depends only on S_t. In the standard case, the hidden Markov model is assumed to be *homogeneous* in the sense that the Markov chain is homogeneous, that is, the transition probabilities are constant over time, as is the conditional distribution of $\mathbf{Y}_t$ given S_t. Through the observed process $\{\mathbf{Y}_t : t = 1, \ldots, T\}$, inference can be made about *both* the model parameters and the hidden process which is often of interest in its own right. The inclusion of the unobserved random variables means that hidden Markov models may also be regarded as *missing data models* or *latent variable models*.

Specific features of an observed time series, $(\mathbf{y}_1, \ldots, \mathbf{y}_T)$, can make hidden Markov models particularly attractive. Hidden Markov models are generalisations of mixture models and so also allow for overdispersion, skewness and multi–modality. This makes them useful when the marginal distributions of observables exhibit these traits. Alternatively, in situations where ordered data are believed to have arisen in distinct segments, hidden Markov models can provide a means of capturing any underlying heterogeneity. Examples include DNA sequence data and series with reversible or irreversible "change points", such as the demand for natural gas by industrial users, which can exhibit "jumps" as manufacturers switch between sources of energy. Finally, if a temporal process can only be observed in noise, hidden Markov models provide a means of extracting the signal, for example, in digital communications and speech recognition.

In this project, students will take a Bayesian approach to inference, fitting models using Markov chain Monte Carlo methods. Beyond this, there is flexibility over the precise topic of research. Possible areas for exploration include

- Development and comparison of different hidden Markov models for a particular real-world data set,
- Determination of an appropriate or "useful" number of states,
- Incorporation of time-varying covariate information in the hidden or observed process,
- Incorporation of Markovian dependence in the observed process,
- Parallel tempering to facilitate movement between posterior modes,
- So-called infinite hidden Markov models which are a non-parametric Bayesian approach to hidden Markov models.

Essential prior modules

- MATH2707: Markov Chains II,
- MATH2647: Probability II,
- MATH2711: Statistical Inference II,
- MATH2697: Statistical Modelling II.

Essential companion modules

- MATH3411: Advanced Statistical Modelling,
- MATH3421: Bayesian Computation and Modelling,
- MATH3251: Stochastic Processes III.

Suggested resources

The following resources provide references, examples and a comprehensive treatment of inference for hidden Markov models:

- Frühwirth-Schnatter, S. (2006). *Finite Mixture and Markov Switching Models*. Springer.
- Scott, S. L. (2002). Bayesian Methods for Hidden Markov Models. *Journal of the American Statistical Association*, **97**, 337–351.
- Zucchini, W., MacDonald, I. L. and Langrock, R. (2016). *Hidden Markov Models for Time Series: An Introduction Using R*, Second Edition. CRC Press.

Supervisor

Sarah Heaps (sarah.e.heaps@durham.ac.uk)