Durham University
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Michaelmas 2023

Topics in Combinatorics IV, Homework 4 (Week 4)

Due date for starred problems: **Friday, November 3 , 6pm**.

4.1. Construct a bijection between all set partitions of $[n]$ and those set partitions of $[n+1]$ that do not contain consequent numbers in one block.

4.2. ($\star$) Show that the number of non-crossing set partitions of $[n]$ with k blocks is equal to the Narayana number $N(n,k)$.
Hint: you may try different models of Catalan numbers we considered in lectures.

4.3. Show the symmetry of Narayana numbers: $N(n,k) = N(n, n-k+1)$.

4.4. A *star graph* is a graph whose all vertices except for one are leaves (i.e., it consists of one vertex connected to every other vertex).

(a) Let c_n be the number of star graphs on n labeled nodes (the graph is not embedded, i.e. it only matters which vertex is connected to which). Compute c_n for every $n \geq 1$.

(b) Show that the exponential generating function $c(x)$ of the sequence (c_n) is

$$c(x) = xe^x - \frac{x^2}{2}$$