Analysis IV Extra Reading Road Map

Durham University, 2009/10

The three sources of the 4H extra reading material for Analysis can be found on the DUO webpage of Analysis IV.

The core material is Simmons. You should understand Theorem A (Weierstrass Approximation Theorem) of Section 35 and Theorems A and B (Real and Complex Stone-Weierstrass Theorem) of Section 36 and you should work through the proofs. If you are not familiar with Hausdorff spaces, you can replace them by the interval $[a, b]$, which is a special case. Sections 37 and 38 are not relevant for us.

For the proof of Theorem A in Section 35 above, we need the concept of "uniform continuity" - the relevant facts are in 7.1.7 and 7.1.8 in Haggarty. Recall that the canonical metric on $C[a, b]$ is given by $d(f, g) = \|f - g\|_\infty = \max_{x\in[a,b]} |f(x) - g(x)|$. Try also to find counterexamples if you drop any of the assumptions of the theorem.

Hints for Section 36: The closure of a set A in a metric space (X, d) is $\bar{A} = \{x \in X \mid \exists x_n \in A \text{ with } x = \lim x_n\}$. A (sub)algebra $A \subset C[a, b]$ is a subset which is closed under multiplication, multiplication by real numbers and addition (i.e., if $f, g \in A$ and $c \in \mathbb{R}$, then $f + g, fg, cf \in A$). Such an algebra A *separates points*, if for every pair $x, y \in [a, b]$ of distinct points there is a function $f \in A$ such that $f(x) \neq f(y)$. You will come across the notion of a (sub)lattice - this is a subset $A \subset C[a, b]$ such that if $f, g \in C[a, b]$ then $f \vee g, f \wedge g \in A$ (these operations are defined in Simmons). You also need the Heine-Borel covering property of compactness (please see Hairer/Wanner).

The first paragraph steering towards Theorem B which mentions Morera's theorem (see p. 160 in Simmons) is optional. But try to understand what has to be modified to get the result in the complex case.

If you fail to understand some paragraph of the reading material, my advise is to skip it at first reading and to come back to it later.