Analysis III/IV (Math 3011, Math 4201)

Exercise Sheet 13 18.02.2010

1. Let $\varphi : (0,\infty) \times (0,2\pi) \times (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}^3$ be defined by

$$\varphi(r,\alpha,\beta) = (r\cos\alpha\cos\beta, r\sin\alpha\cos\beta, r\sin\beta).$$

   Calculate the pullback $\varphi^*(xdy)$.

2. Let ω_1, ω_2 be differential forms, defined on some open set $U \subset \mathbb{R}^n$. Assume that ω_1 is closed and ω_2 is exact. Show that $\omega_1 \wedge \omega_2$ is closed and exact.

3. Let $A_i \subset \mathbb{R}^n$ be a countable sequence of sets of measure zero. Show that the union $\cup_i A_i$ is, again, a set of measure zero.