Analysis III/IV (Math 3011, Math 4201)

Exercise Sheet 10 28.01.2010

1. Show that $\frac{\partial\varphi}{\partial u_1} \times \frac{\partial\varphi}{\partial u_2} : U \to \mathbb{R}^3$ of a surface parametrisation $\varphi : U \to S \subset \mathbb{R}^3$ is perpendicular to the surface, i.e., for every $x = \varphi(u) \in S$ and every curve $c : (a.b) \to S$ with $c(0) = x$ we have

$$\frac{\partial\varphi}{\partial u_1} \times \frac{\partial\varphi}{\partial u_2}(u) \perp c'(0).$$

2. Calculate the length of the **cycloid**

$$c : [0, 2\pi] \to \mathbb{R}^2, \quad c(t) = (t - \sin t, 1 - \cos t).$$

(The answer should be 8.)

3. Let $\omega \in \Omega^1(\mathbb{R}^2\backslash\{0\})$ be given by

$$\omega = -\frac{y}{x^2 + y^2}\, dx + \frac{x}{x^2 + y^2}\, dy.$$

Let $r > 0$ and $c : [0, 2\pi] \to \mathbb{R}^2$ be defined as $c(t) = (r \cos t, r \sin t)$. Show that ω is closed and calculate

$$\int_c \omega.$$

Decide whether it is possible that there is a function $f \in C^\infty(\mathbb{R}^2\backslash\{0\})$ satisfying $\omega = df$.

4. Show that the form $\omega = 2xy^3 dx + 3x^2y^2 dy$ is closed and compute $\int_c \omega$, where c is the arc of the parabola $y = x^2$ from $(0, 0)$ to (x, y).