DURHAM UNIVERSITY — Department of Mathematical Sciences COLLECTION 2015 Name: College:

Analysis MATH1051

Name:	College:	
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Time allowed: 45 minutes. Answer all questions . Use of electronic calculators is forbidden. You should explicitly refer to (but not necessarily write out the statements of) any of the main results of the course that you use.

1. Evaluate the following limits.

(a) (15 marks.)
$$\lim_{n \to \infty} \frac{\sqrt{n-2} - \sqrt{4n+7}}{\sqrt{n}}$$

 $4 \cdot 10^k - 3 \cdot 10^{2k}$

(b) (10 marks.) $\lim_{k \to \infty} \frac{4 \cdot 10^k - 3 \cdot 10^{2k}}{10^{k-1} + 2 \cdot 10^{2k-1}}.$

- 2. (a) (5 marks.) Give an example of a convergent sequence which is not monotone increasing and not monotone decreasing.
 - (b) (5 marks.) Using quantifiers, give the precise logical formulation that a sequence (x_n) does not converge to the value x^* .
 - (c) (15 marks.) Let (a_n) and (b_n) be two convergent sequences with $a_n \to a$ and $b_n \to b$, as $n \to \infty$. Give an (ϵ, N) -proof that $(a_n + b_n)$ is also convergent with $a_n + b_n \to a + b$.

- 3. Calculate the following expressions explicitly.
 - (a) (10 marks.) $f^{-1}([0,1))$ for $f: \mathbb{R} \to \mathbb{R}, f(x) = \sin x$.
 - (b) (15 marks.) $\inf(g)$ and $\sup(g)$ for $g: \mathbb{R} \to \mathbb{R}$ with $g(x) = (2 + 3x^2 \sin x)/(3 + x^2)$.

- 4. (a) (5 marks.) State the Bolzano-Weierstrass Theorem.
 - (b) (5 marks.) Let $a, b \in \mathbb{R}$ be two real numbers with a < b. Give the contrapositive statement to

A: If $f:[a,b] \to \mathbb{R}$ is a continuous function, then f is bounded.

- (c) (5 marks.) Give a reformulation of the unboundedness of a function $f : [a, b] \to \mathbb{R}$ in terms of a sequence (x_n) with particular properties.
- (d) (10 marks.) Use (c) and Bolzano-Weierstrass to prove the contrapositive statement to A.