Problem 1. Let $M$ and $N$ be manifolds. Let $d$ be a metric on $N$. Show that the collection of sets of the form

$$W(f, K, \varepsilon) := \{ f \in C(M, N) \mid d(f(x), g(x)) < \varepsilon \text{ for all } x \in K \}$$

where $K \subseteq M$ is compact and $\varepsilon > 0$ is a basis for the compact open topology on $C(M, N)$.

Problem 2. Let $M$ be a manifold. Show that $\text{Homeo}(M)$ is locally contractible at each $f \in \text{Homeo}(M)$ if and only if it is locally contractible at $\text{Id}$.

Problem 3. For $i \in \mathbb{N}$, let $B_i$ denote the ball of radius $\frac{1}{3}$ centred at $(i, 0) \in \mathbb{R}^2$. Define $M := \mathbb{R}^2 \setminus \bigcup_i B_i$.

Let $h_i \in \text{Homeo}(M)$ be a homeomorphism which is the identity outside the disc of radius 1 centred at $(i + \frac{1}{2}, 0)$, and which maps $B_i$ to $B_{i+1}$ and vice versa. Why does such a homeomorphism exist?

Show that $h_i$ is not homotopic to the identity for any $i$, but $\{h_i\}$ converges to the identity in the compact open topology on $\text{Homeo}(M)$.

Conclude that $\text{Homeo}(M)$ is not locally contractible, nor locally path connected.