Problem 1. Let $X$ and $Y$ be compact metric spaces with $X \times \mathbb{R}$ homeomorphic to $Y \times \mathbb{R}$. Then $X \times S^1$ is homeomorphic to $Y \times S^1$.

Hint: let $h: X \times \mathbb{R} \to Y \times \mathbb{R}$ be a homeomorphism, and consider the two product structures on $Y \times \mathbb{R}$, the intrinsic one and the one coming from $h(X \times \mathbb{R})$. Use a push-pull construction (repeated infinitely many times) to create a periodic homeomorphism $H: X \times \mathbb{R} \to Y \times \mathbb{R}$, i.e. for some $p \in \mathbb{R}$, $H(x, t) = H(x, t + p)$ for all $t \in \mathbb{R}, x \in X$. 