October 2013

Introduction to Computer Algebra

Department of Mathematical Sciences, Durham University

Computer algebra software can help you learning and doing mathematics. It may be used to reduce the tedium of extended calculations, to verify the correctness of hand calculations and also for exploration of a new topic. It is *not* a substitute for mathematical thinking. Think about it: if a computer program could solve all mathematical problems, there would be no jobs for people with maths degrees. Also take note that computer algebra software usually, but *not always*, gets the right answer.

Computer algebra is embedded in many modules at Durham University, and is used by lots of lecturers for demonstrations and to ease calculations. Also, all students studying a Mathematics degree have to do a project in their final year, and by learning how to use computer algebra at an early stage you are more likely to understand it practically and benefit from what it has to offer before you get to your final year.

Getting and running the software

There are several general purpose computer algebra systems available, some for free (e.g. Maxima) and others for a substantial price (e.g. Mathematica). At Durham University we have a site licence for Maple, which is available to use on all IT service networked computers. If you wish to purchase Maple for your own computer you may do so at the substantially discounted price of £15 from the CIS Help Desk. Instructions on how to start Maple on CIS machines can be found at

http://www.dur.ac.uk/cis/maple/

Basics on using Maple and this worksheet

Current versions of Maple have a 'document mode' and a 'worksheet mode'; we will use the latter for this tutorial. Get to the stage where you have a worksheet window with a '>' prompt on your screen. Type:

1+2;

(don't forget the semicolon), hit enter and see what you get. Now try

?sqrt

So now you know what to do when you're lost! A "?" gets you into help and not only gives you assistance on Maple commands, but also hints on Mathematical definitions and other uses.

The exercises in this handout (keep it — you may need it later) will guide you through some of Maple's basic commands. You should *try* each example (text in **bold font** should be typed in *exactly* as shown at first, paying particular attention to CapitalLetters and punctuations) and *understand* what the commands are doing. Later exercises build on earlier ones, so if you get stuck, reviewing previous questions often helps.

Computing with Numbers

1.1 Find out and explain what these Maple commands do:

```
2^4 + 3*7;
1/3! + 1/5;
sin(Pi/4);
I^2;
yy := (125*24)/(74*15);
```

1.2 Note the output of the last command. Now try:

```
evalf(exp(1));
evalf[30](yy);
```

1.3 Compute the first 250 digits of π and the first 20 (significant) digits of e^{-20}.

1.4 Compute 99^{198}, 100! and $\tan(\pi/2)$ in Maple. How are the results different from what you would get with a calculator?

Symbolic Computation

2.1 Maple can perform symbolic computations:

```
z := (x+a)*(x-a) + yy;
expand(z);
```

2.2 Explain what these Maple commands do:

```
f := x -> x^3*exp(x);
42 + f(x^2);
```

Plotting

3.1 Explain what the following commands do:

```
plot( sin(x), x=0..3 );
plot( abs(x), x=-3..3 );
plot( x*exp(x), x=-2..2, colour=blue );
plot( [f(x), f(-x), abs(f(x))], x=-1..1 );
```

3.2 Given $f(x) = x^2 e^x$, plot $f(x)$, $f(x) + f(-x)$ and $f(x) - f(-x)$ for $x \in [-1, 1]$. Repeat for $g(x) = \sin(x + 1)$ and $h(x) = 3x^5 + 2x^2$.

3.3 Plot $f(x) = x - |x|$ and $g(x) = x/|x|$.

3.4 A three-dimensional plot:

```
plot3d( sin(x+2*y), x=-3..3, y=-3..3, axes=box );
```

Calculus

4.1 Explain what these do:

diff(sin(x)*cos(x), x);

diff(sin(2*x), x$3);

int(1/x, x);

int(ln(x), x=0..10);

int(exp(-3*x), x=0..infinity);

4.2 Compute the (indefinite) integral of $(x^6 + x^3 + 1)\sin(1 + x)$.

Linear algebra

5.1 Try:

u := Vector([1,2,3]);

v := Vector([3,5,7]);

u.v;

A := Matrix([[1,0,0], [0,2,0], [-1,0,3]]);

A.u;

5.2 For more "fancy" operations, we need to load a "package":

with(LinearAlgebra);

Now try:

CrossProduct(u,v);

%.v;

MatrixInverse(A);

%.A;

Sequences, series and products

6.1 Explain what these do:

seq(ithprime(j),j=1..100);

sum(1/k!,k=1..20);

product(n^2,n=1..5);

seq(evalf[17](sum(1/k!,k=0..n)),n=1..20);

6.2 Explain what these (in particular the command **eval**) do:

```
diff(x^5+3*x^4,x$3);
expand( exp(x^2)*diff( exp(-x^2), x$6 ) );
eval( diff(exp(-x^2),x$3), x=0 );
seq( eval( diff(exp(-x^2),x$n), x=0 ), n=1..23 );
```

Questions and more information

The best way to learn a computer algebra system is to use it. There is an enormous amount of material on Maple, Mathematica, Maxima etc. on the web, so google around. There is also some additional information (e.g. on Maxima and Maple) on the web page of this course,

http://maths.dur.ac.uk/users/kasper.peeters/intro_computer_algebra.html

For any questions on computer algebra, now or during the year, please contact me.

kasper.peeters@durham.ac.uk