Durham University

North-East and Midlands Stochastic Analysis Seminar

supported by *the London Mathematical Society and Isaac Newton Institute for Mathematical Sciences*

Organisers: *Zdzislaw Brzezniak (York), Horatio Boedihardjo (Warwick) David Elworthy (Warwick), Chunrong Feng (Durham), Maximiliano Gubinelli (Oxford), Zhongmin Qian (Oxford), Roger Tribe (Warwick), Huaizhong Zhao (Durham)*

Programme (Venue: MCS2068, Department of Mathematical Sciences)

Monday 8 July

14:00-15:00: **Martin Barlow (British Columbia)**

Sleeping Beauty meets Kolmogorov

15:00-15:30: Break

15:30-16:30: **Alpar Meszaros (Durham)**

From N-player differential games to the master equation in mean field games

16:30-17:30: **Hiroshi Kawabi (Keio)**

A graph discretized approximation of diffusions with drift and killing on a complete Riemannian manifold

Tuesday 9 July

9:00-10:00: **Michael Rockner (Bielefeld)**

Nonlinear Fokker-Planck equations and nonlinear Markov processes: The 2D vorticity Navier-Stokes equation

10:00-11:00: **Ilya Pavlyukevich (Jena)**

Stochastic energy-balance model with a moving ice line

11:00-11:30: Break

11:30-12:30: **Mickael Chekroun (Weizmann)**

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14:00-15:00: **Jian Song (Shandong)** (Zoom: 6974011449)

Stochastic partial differential equations associated with Feller processes

15:00-16:00: **Xiangchan Zhu (CAS)** (Zoom: 6974011449)

Non-unique ergodicity for deterministic and stochastic 3D Navier--Stokes and Euler equations

If you have any queries, please contact Chunrong Feng (chunrong.feng@durham.ac.uk) or Huaizhong Zhao (huaizhong.zhao@durham.ac.uk).

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Titles and Abstracts

Martin Barlow (University of British Columbia)

Title: Sleeping Beauty meets Kolmogorov

Abstract: *The Sleeping Beauty problem was formulated in the philosophy literature around 2001, and probes the question of inference when a random experiment may give rise to multiple observers. At first sight it may appear to be a simple exercise in Bayes' theorem (like the Monty Hall problem), but I assert that it is more subtle than that. I will show that the problem can be handled by Kolmogorov's axioms, but that all solutions lead to problematic or paradoxical conclusions.*

Alpar Meszaros (Durham University)

Title: From N-player differential games to the master equation in mean field games

Abstract: *The theory of mean field games (MFG) was initiated around 2006 by two groups, Lasry—Lions and Huang—Malhamé—Caines. Their main goal was to characterize limits of Nash equilibria of N-player stochastic differential games, as the number of agents tends to infinity. In this talk we will present some recent progress in the theory, with a particular focus on the so-called master equation, proposed by Lions. This is a PDE of hyperbolic type set on the space of probability measures, which encapsulates all the information about the underlying games. We will be focusing on a class of data that fulfill the so-called displacement monotonicity condition — rooted in the theory of optimal transport — which ensures global well-posedness of the corresponding master equations.*

Hiroshi Kawabi (Keio University)

Title: A graph discretized approximation of diffusions with drift and killing on a complete Riemannian manifold

Abstract*: In this talk, we present a graph discretized approximation scheme for diffusions with drift and killing on a complete Riemannian manifold M. More precisely, for a given Schrödinger operator with drift on M having the form* $\mathscr{A} = -\Delta - b + V,$ *we introduce a family of discrete time random walks in the flow generated by the drift b with killing on a sequence of proximity graphs, which are constructed by partitions cutting M into small pieces. As a main result, we prove that the drifted Schrödinger semigroup* $\{\mathrm{e}^{-t\mathscr{A}}\}_{t\geq 0}$ *is approximated by discrete semigroups generated by the family of random walks with a suitable scale change. This result gives a finite dimensional summation approximation of a*

Feynman-Kac type functional integral over M. Furthermore, when M is compact, we also obtain a quantitative error estimate of the convergence.

This talk is based on a joint work with Satoshi Ishiwata (Yamagata University) and the full paper can be found on https://doi.org/10.1007/s00208-024-02809-9 (online-first article in Mathematische Annalen).

Michael Rockner (Bielefeld University)

Title: Nonlinear Fokker-Planck equations and nonlinear Markov processes: The 2D vorticity Navier-Stokes equation

Abstract: *Since the middle of last century a substantial part of stochastic analysis has been devoted to the relationship between (parabolic) linear partial differential equations (PDEs), more precisely, linear Fokker-Planck-Kolmogorov equations (FPKEs), and stochastic differential equations (SDEs), or more generally Markov processes. Its most prominent example is the classical heat equation on one side and the Markov process given by Brownian motion on the other. This talk is about the nonlinear analogue, i.e., the relationship between nonlinear FPKEs on the analytic side and McKean-Vlasov SDEs (of Nemytskii-type), or more generally, nonlinear Markov processes in the sense of McKean on the probabilistic side. This program has been initiated by McKean already in his seminal PNASpaper from 1966 and this talk is about recent developments in this field. Topics will include existence and uniqueness results for distributional solutions of the nonlinear FPKEs on the analytic side and equivalently existence and uniqueness results for weak solutions of the McKean-Vlasov SDEs on the probabilistic side. Furthermore, criteria for the corresponding path laws to form a nonlinear Markov process will be presented. Among the applications are e.g. porous media equations (including such with nonlocal operators replacing the Laplacian and possibly being perturbed by a transport term) and their associated nonlinear Markov processes. A special emphasis will lie on the 2D Navier-Stokes equation in vorticity form and its associated nonlinear Markov process.*
Joint work with:
Viorel Barbu, Al.I. Cuza University and Octav Mayer Institute of Mathematics of Romanian Academy, Ia.si, Romania Deng Zhang, Shanghai Jiao Tong University
References on which the talk is based:
[1] Barbu, V., RÅNockner, M., Nonlinear Fokker–Planck flows and their probabilistic counterparts, Lecture Notes in Mathematics, Springer 2024+, pp. ix + 214.
[2] Barbu, V., RÅNockner, M., Nonlinear Fokker–Planck equations with fractional Laplacian and McKean–Vlasov SDEs with LÅLevy noise, Probab. Theory Rel. Fields (2024), online.

Ilya Pavlyukevich (The Friedrich Schiller University of Jena)

Title: Stochastic energy-balance model with a moving ice line

Abstract: *In [SIAM J. Appl. Dyn. Sys., 12(4):2068--2092, 2013], Widiasih proposed and analyzed a deterministic one-dimensional Budyko--Sellers energy- balance model with a moving ice-line. In this talk, we extend this model to the stochastic setting and analyze it*

within the framework of stochastic slow-fast systems. We derive the dynamics for the ice line in the limit of a small parameter as a solution to a stochastic differential equation. The stochastic approach enables the study of co-existing (metastable) climate states as well as the transition dynamics between them.

Mickael Chekroun (Weizmann Institute of Science)

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Jian Song (Shandong University)

Title: A graph discretized approximation of diffusions with drift and killing on a complete Riemannian manifold

Abstract: *For the stochastic partial differential equation associated with a Feller process, we obtain Feynman-Kac type of representations for the Stratonovich and Skorohod solutions as well as for their moments. The regularity of the law and the Hölder continuity of the solutions are also studied. The talk is based on joint work with Meng Wang and Wangjun Yuan.*

Xiangchan Zhu (Chinese Academy of Sciences)

Title: Non-unique ergodicity for deterministic and stochastic 3D Navier--Stokes and Euler equations

Abstract: *We establish existence of infinitely many stationary solutions as well as ergodic stationary solutions to the three dimensional Navier--Stokes and Euler equations in the deterministic as well as stochastic setting, driven by an additive noise. The solutions belong to the regularity class $C(\mR;H^{\vartheta})\cap C^{\vartheta}(\mR;L^{2})$ for some ${\vartheta}>0$ and satisfy the equations in an analytically weak sense. Moreover, we are able to make conclusions regarding the vanishing viscosity limit. The result is based on a new stochastic version of the convex integration method which provides uniform moment bounds locally in the aforementioned function spaces. Finally we prove that the solutions to the 3D forced Navier-Stokes equations constructed in [BCCDLS22] satisfy an Lp-in-time version of the celebrated Kolmogorov 4/5 law for behavior of the averaged third order longitudinal structure function along the vanishing viscosity limit.*