Durham University

An outflow equilibrium model for the solar corona

Prof. Anthony Yeates
with Dr Oliver Rice


UK SWSE Mtg, Exeter, 10-Sep-2024

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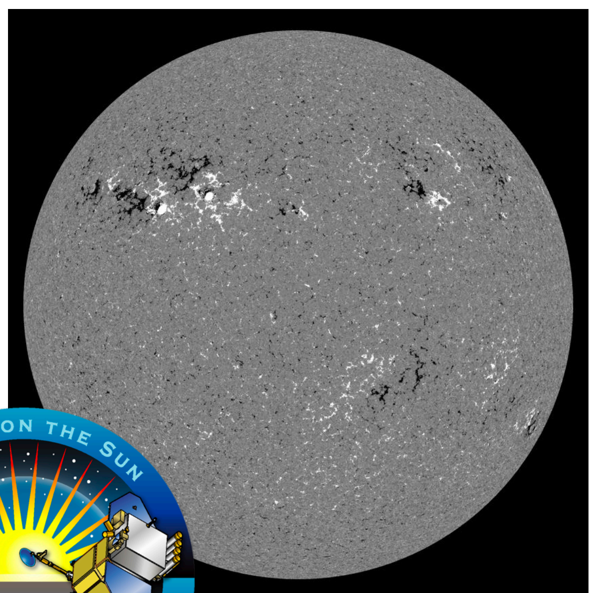
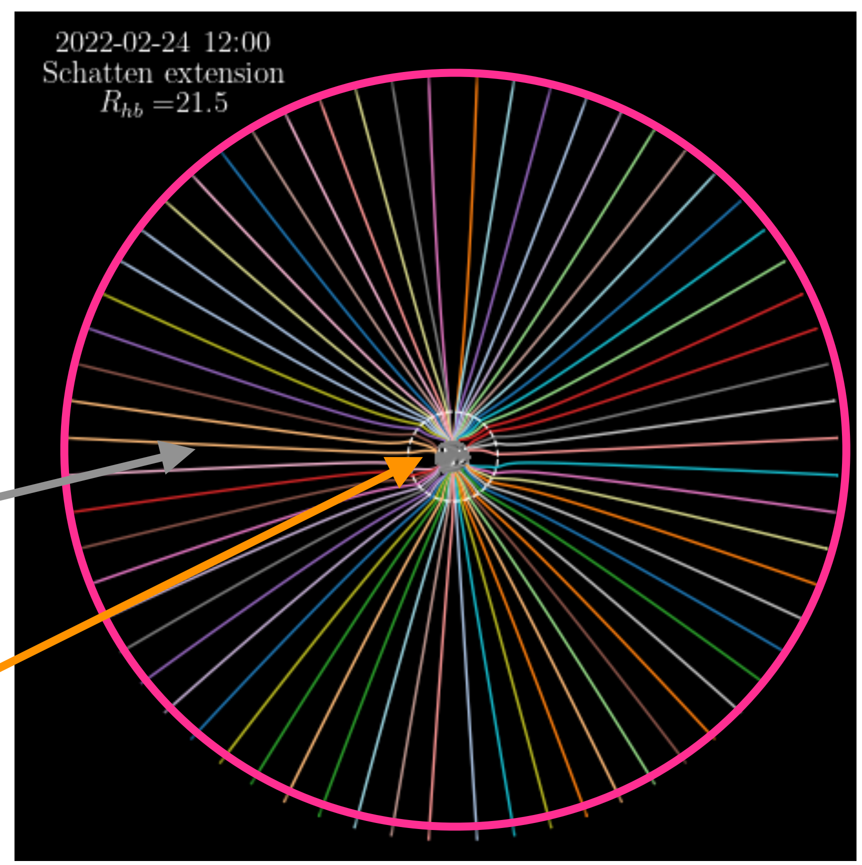
e.g. SWEEP pipeline [SWIMMR/S4]

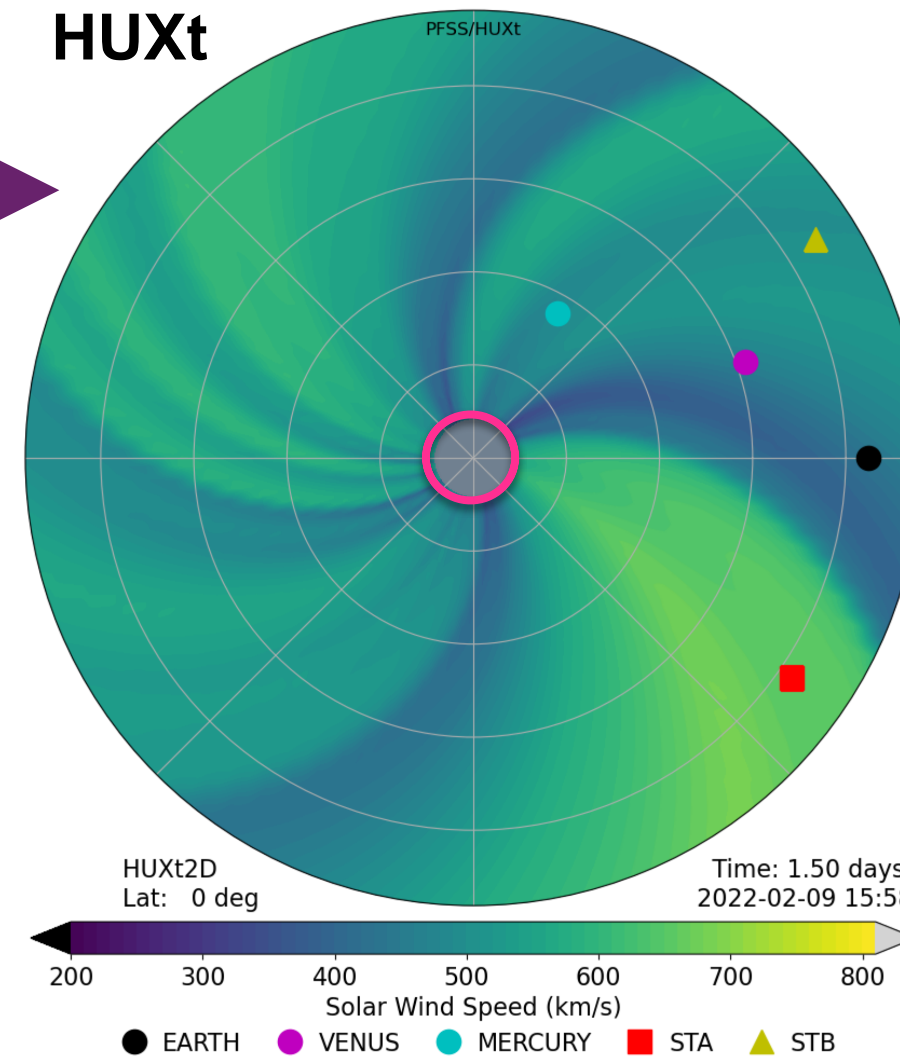
Satellite data



Coronal magnetic model



HUXt

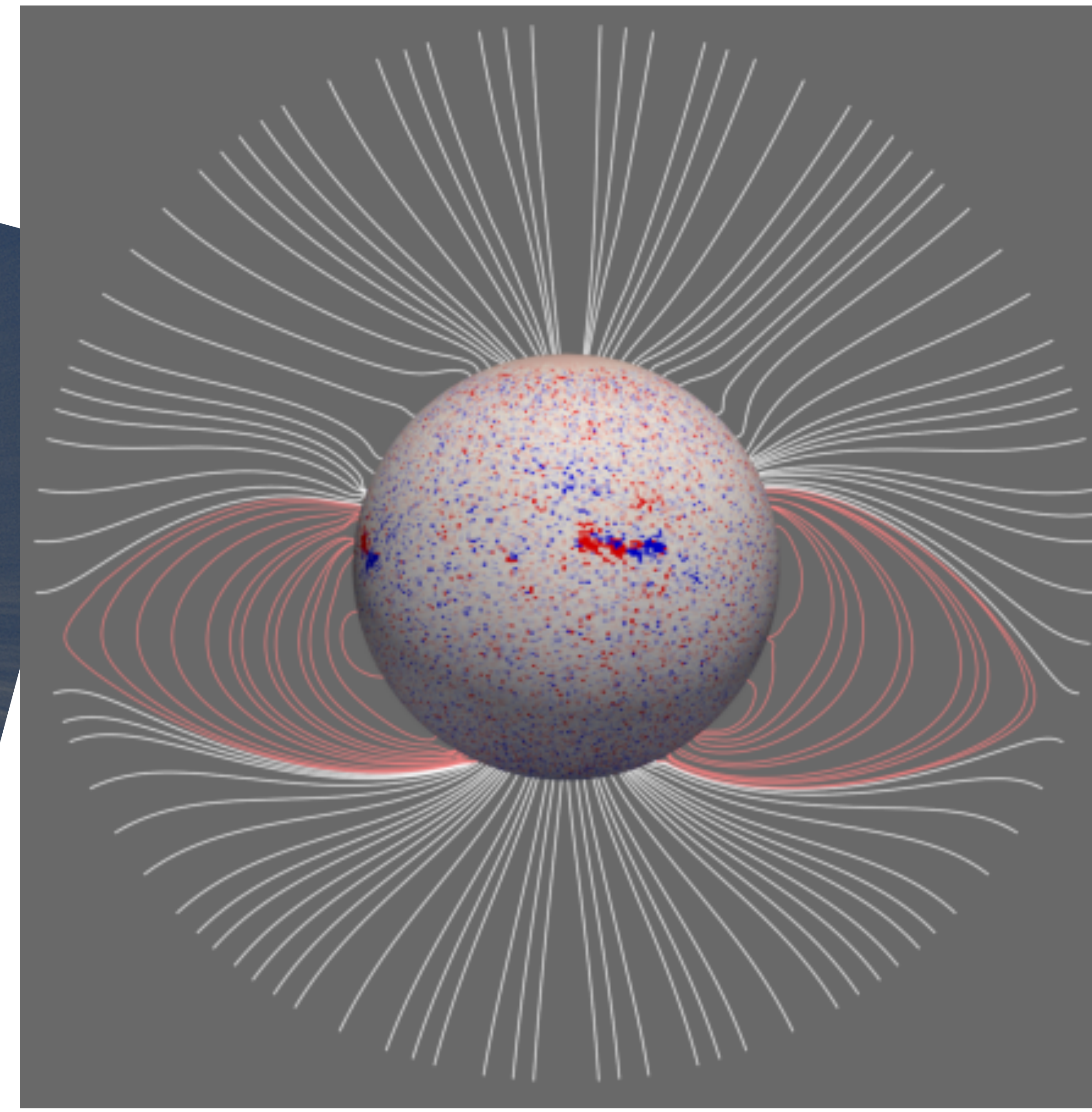


Schatten current sheet model

Main coronal model
(up to $2.5R_{\odot}$)

- Current operational forecasts use the **PFSS (Potential Field Source Surface)** model.

2017-Aug-21



http://www.zam.fme.vutbr.cz/~druck/Eclipse/

pfsspy

- **cf.** full MHD modelling.

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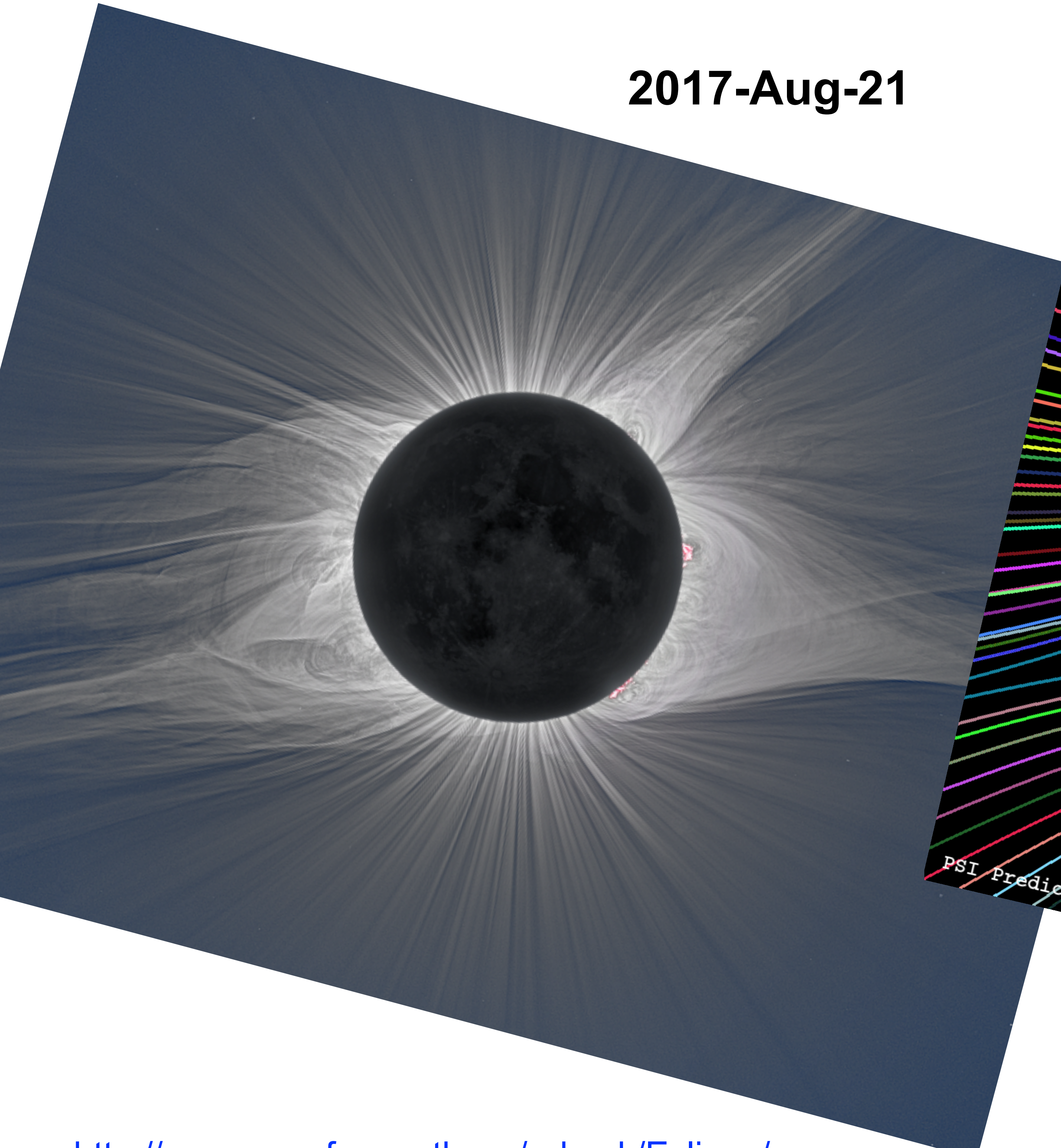

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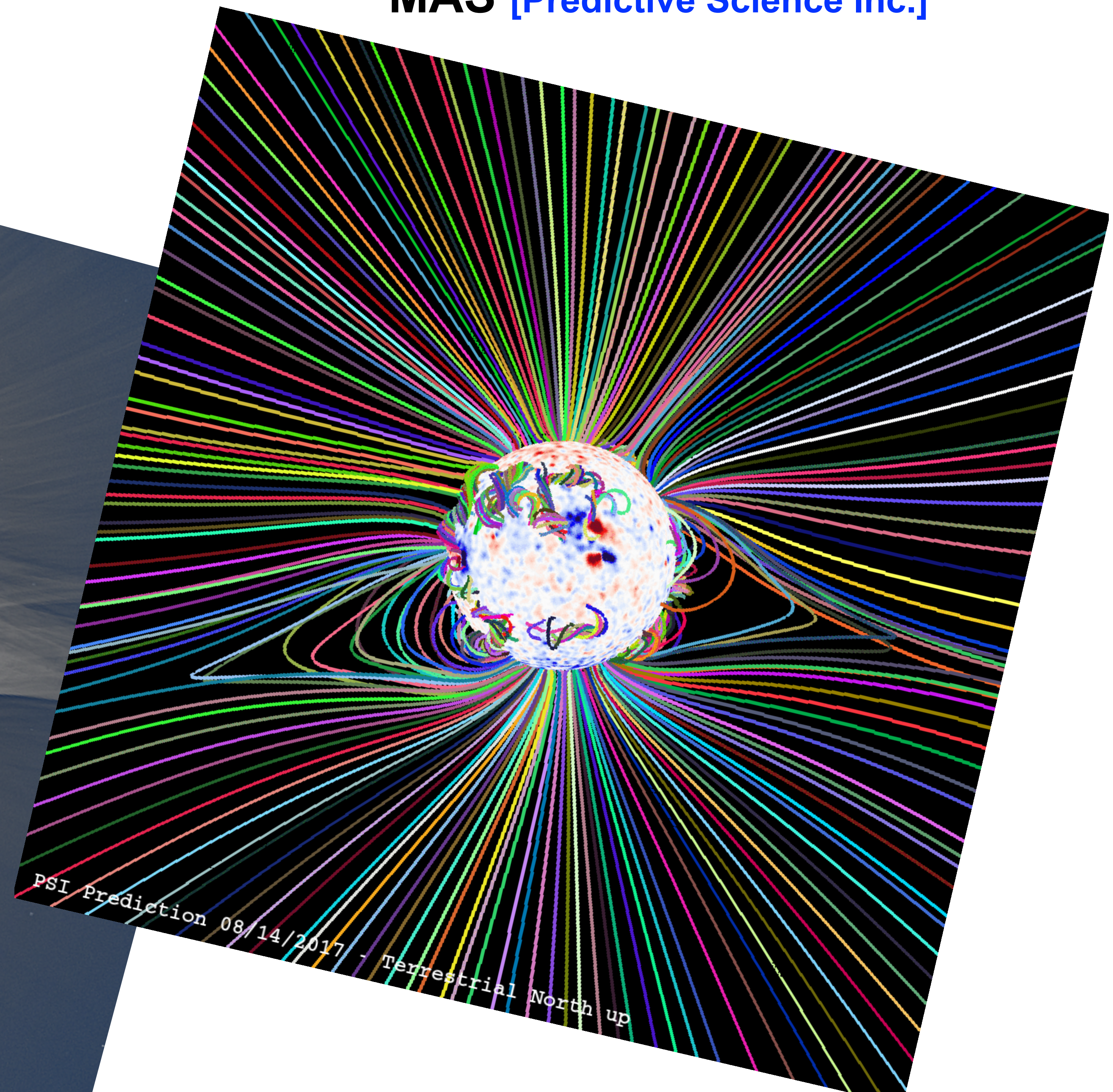
MAS [Predictive Science Inc.]



https://www.predsci.com/corona/aug2017eclipse/

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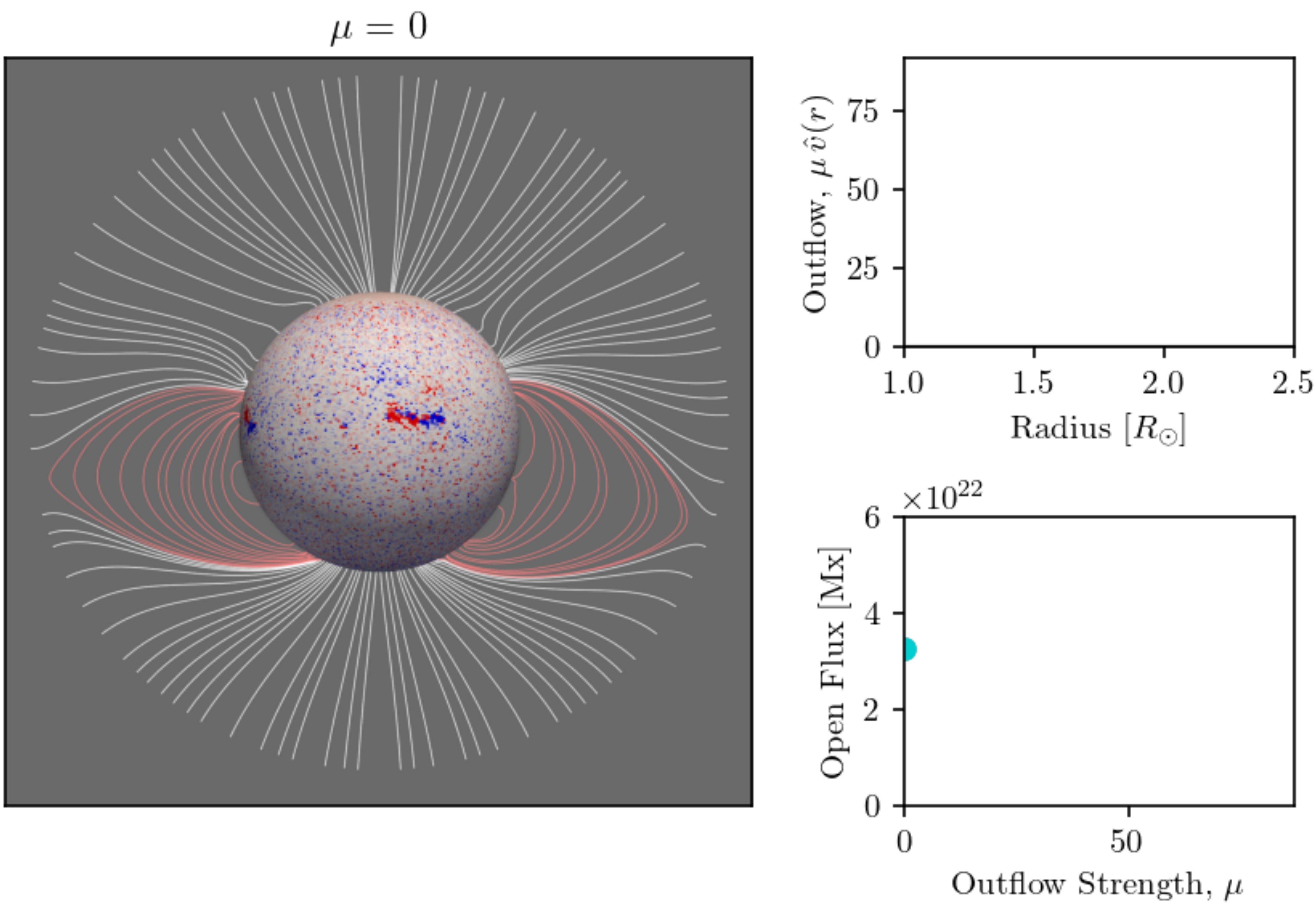

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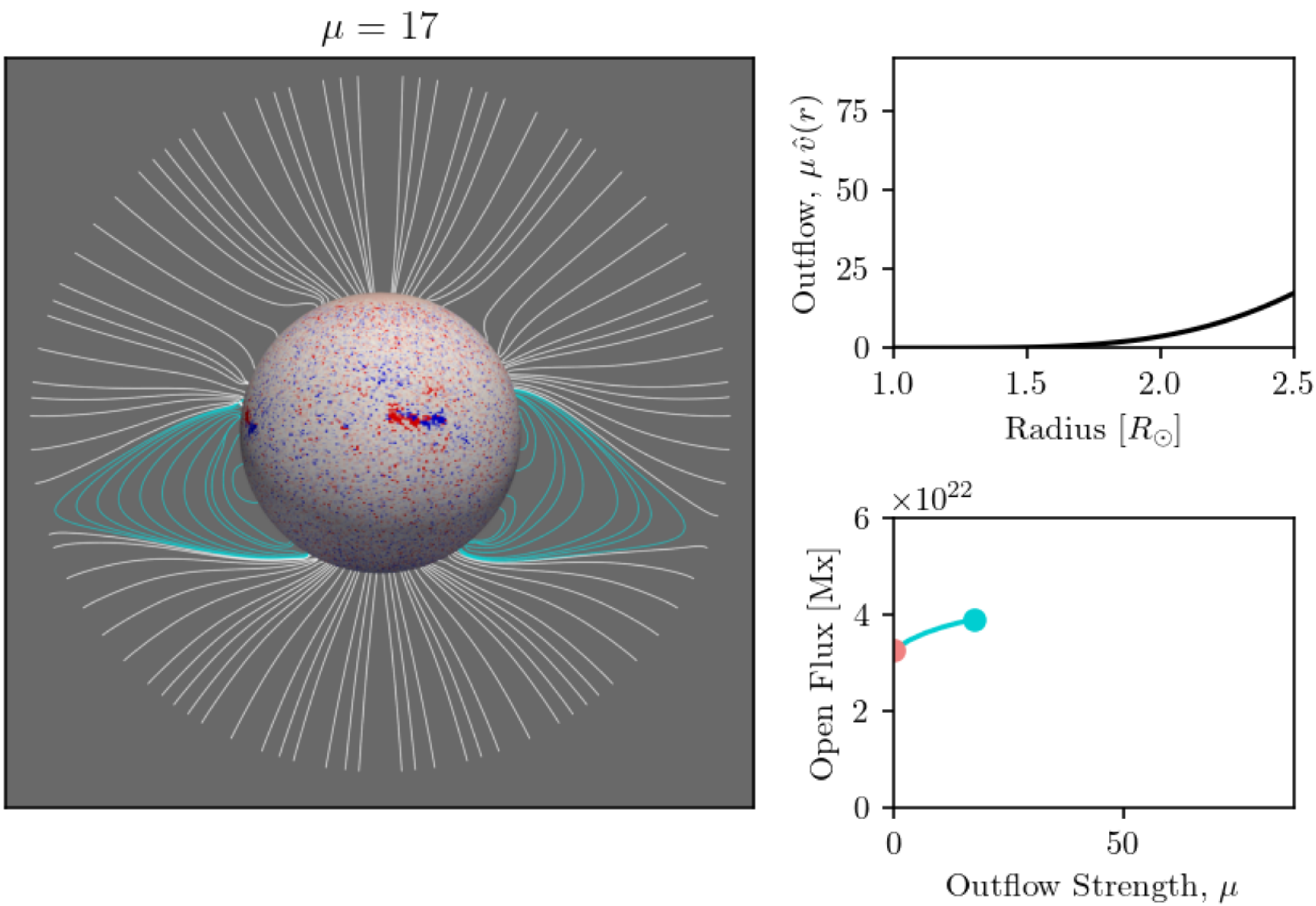

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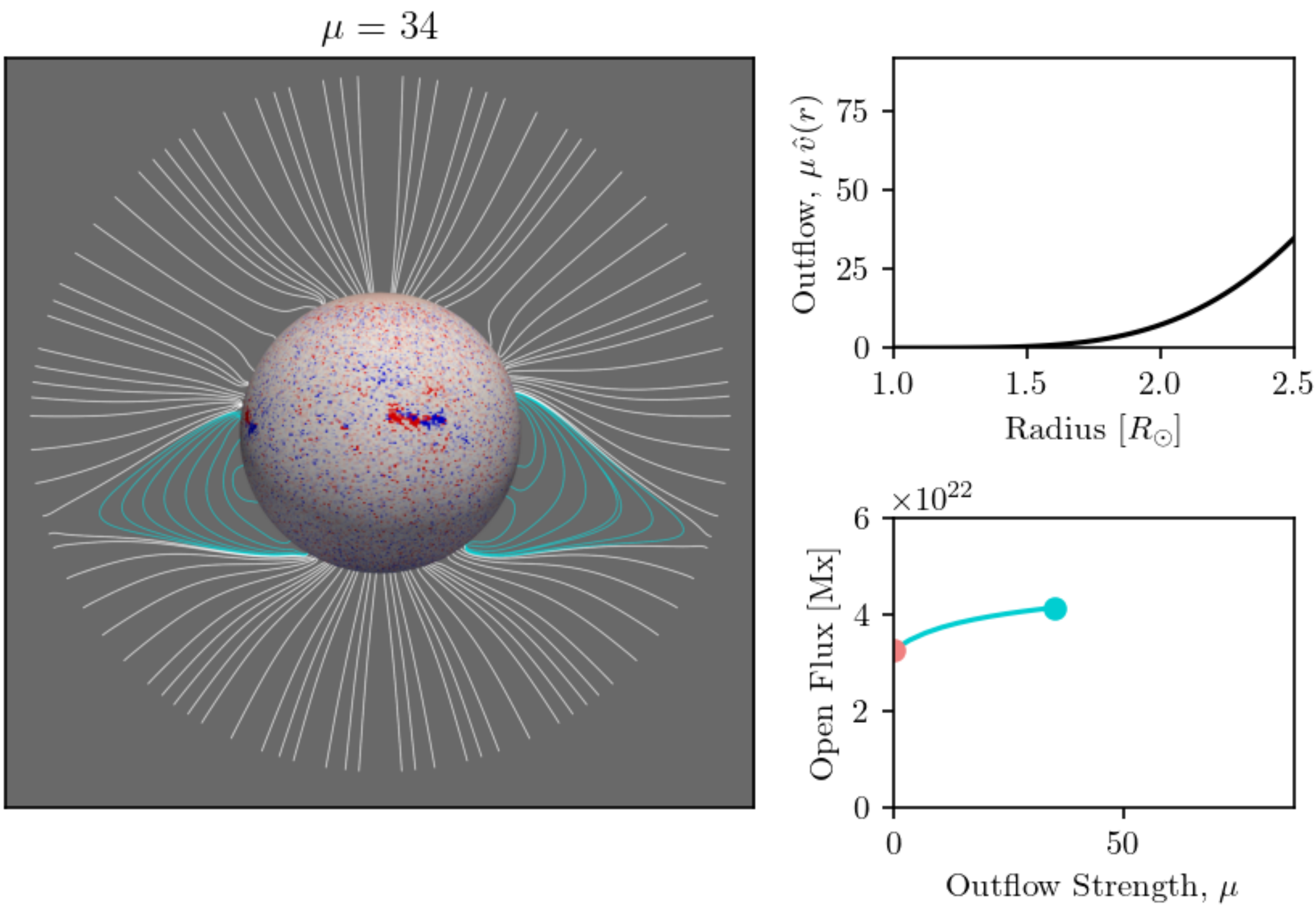

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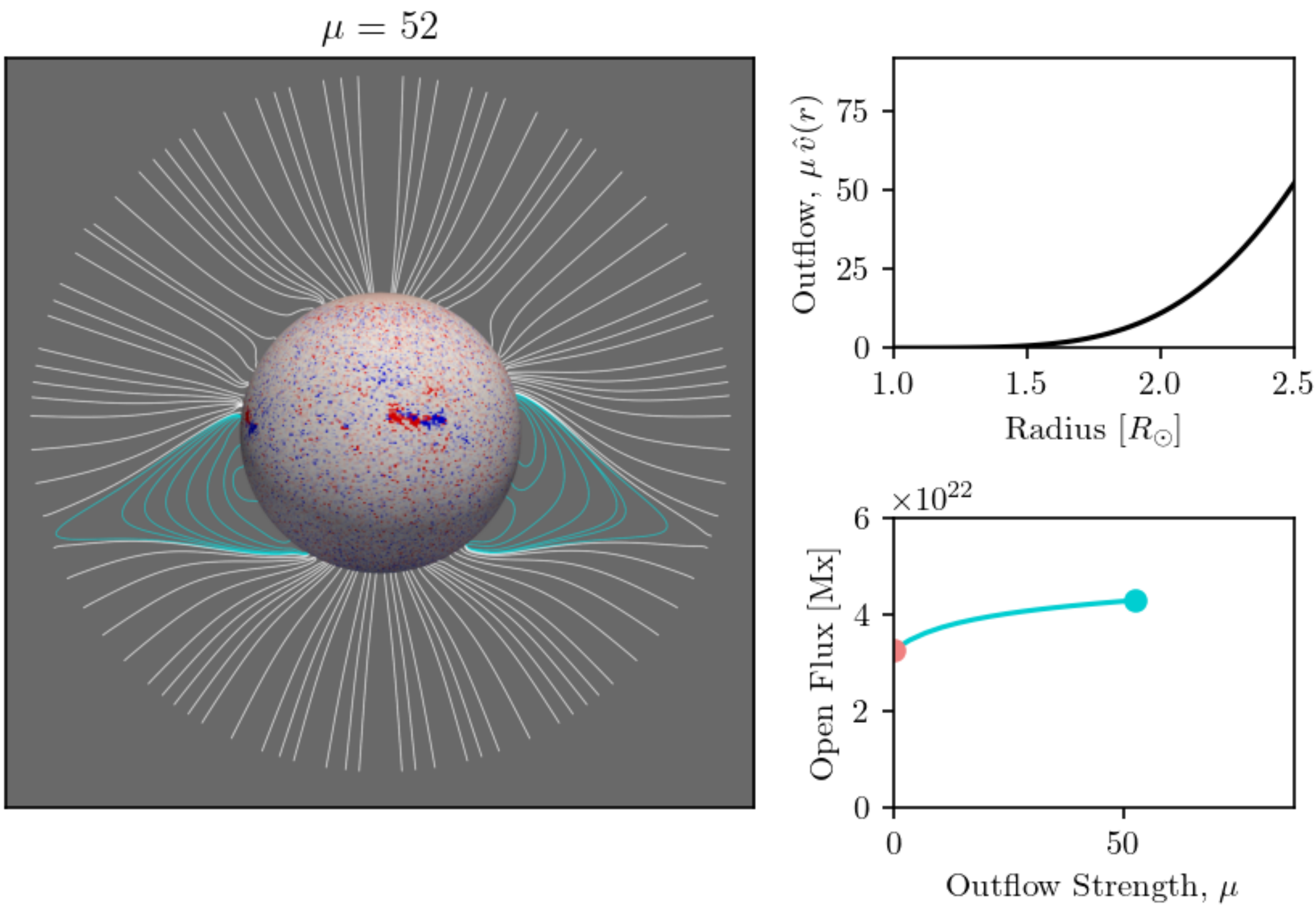

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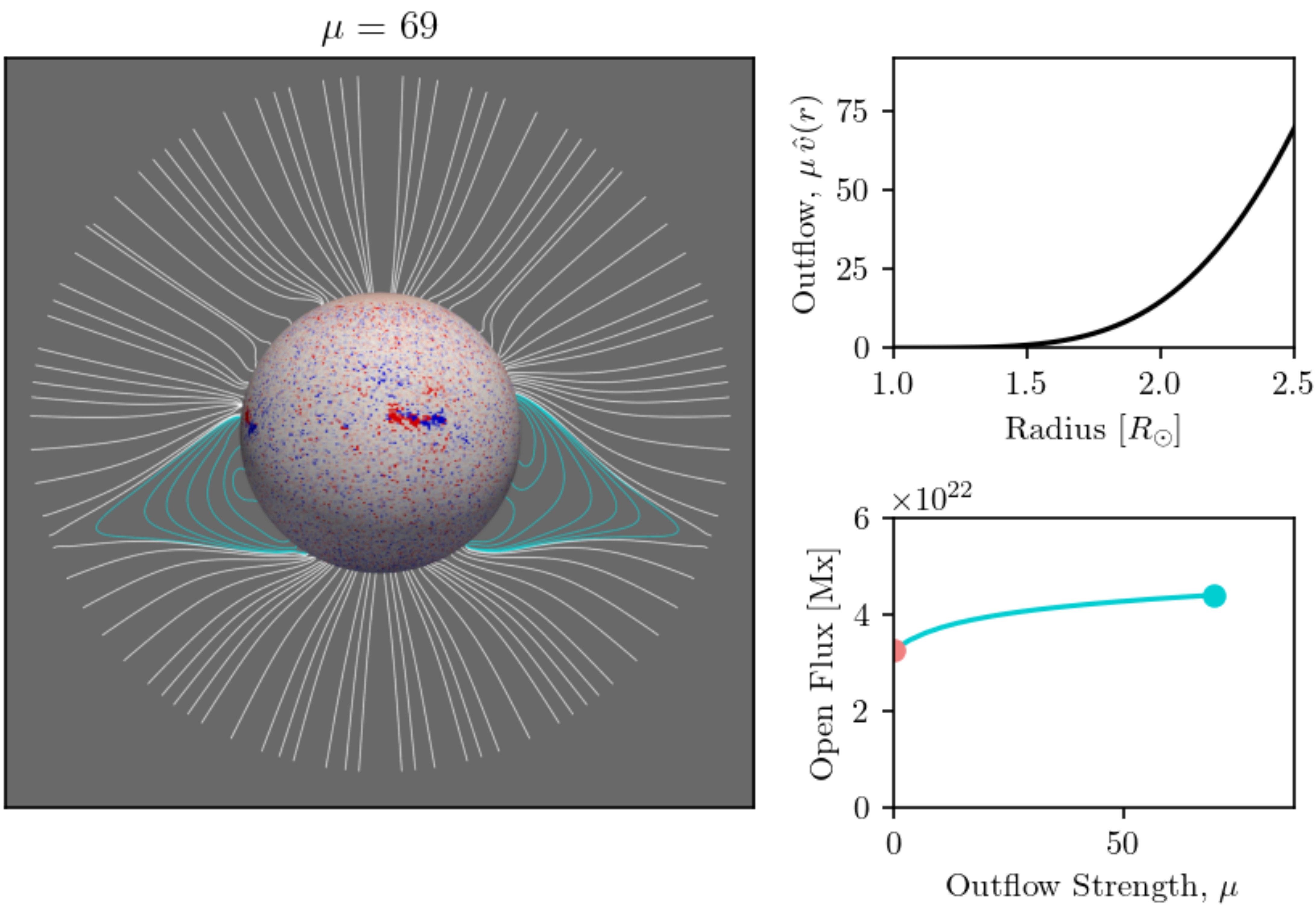

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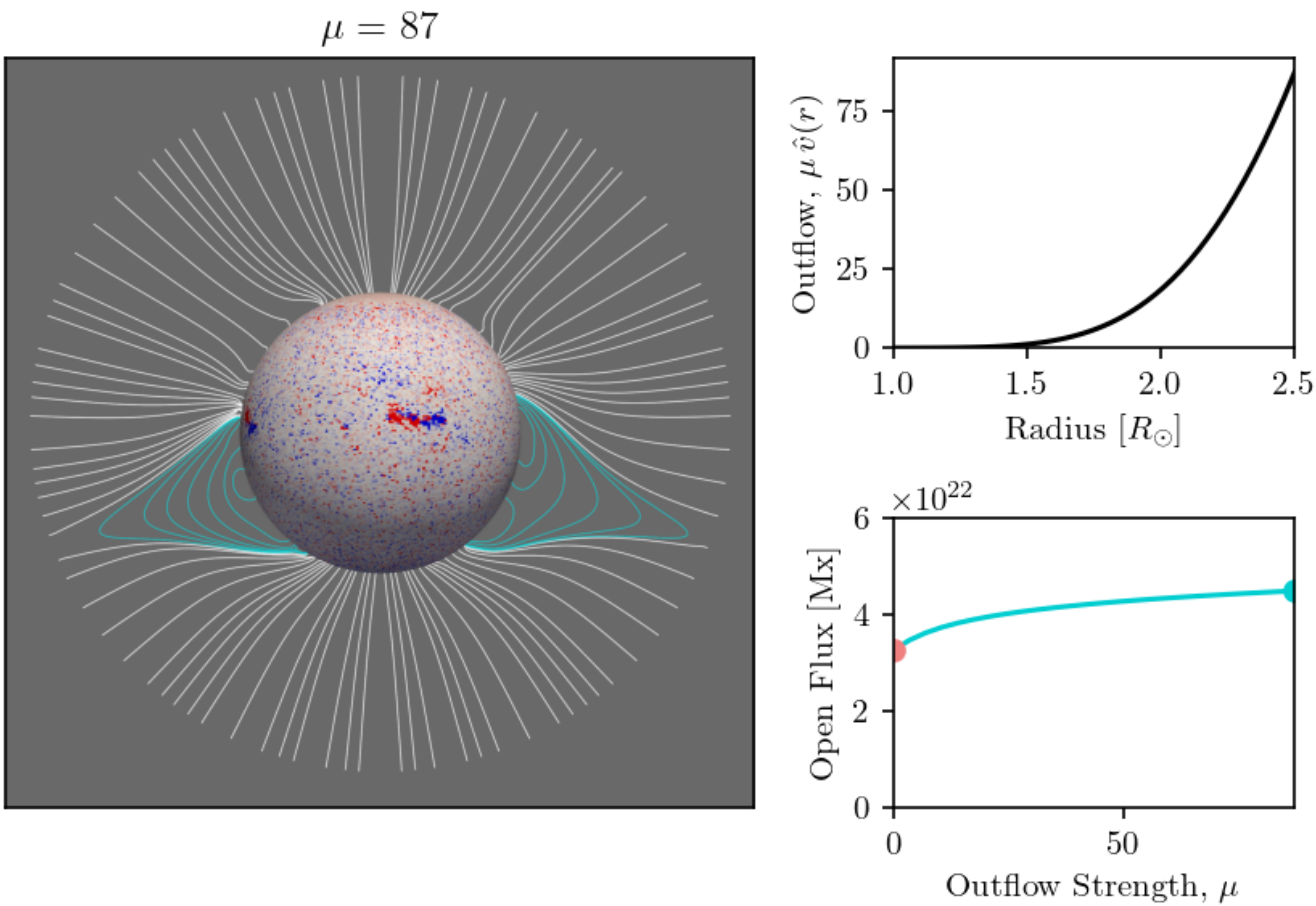

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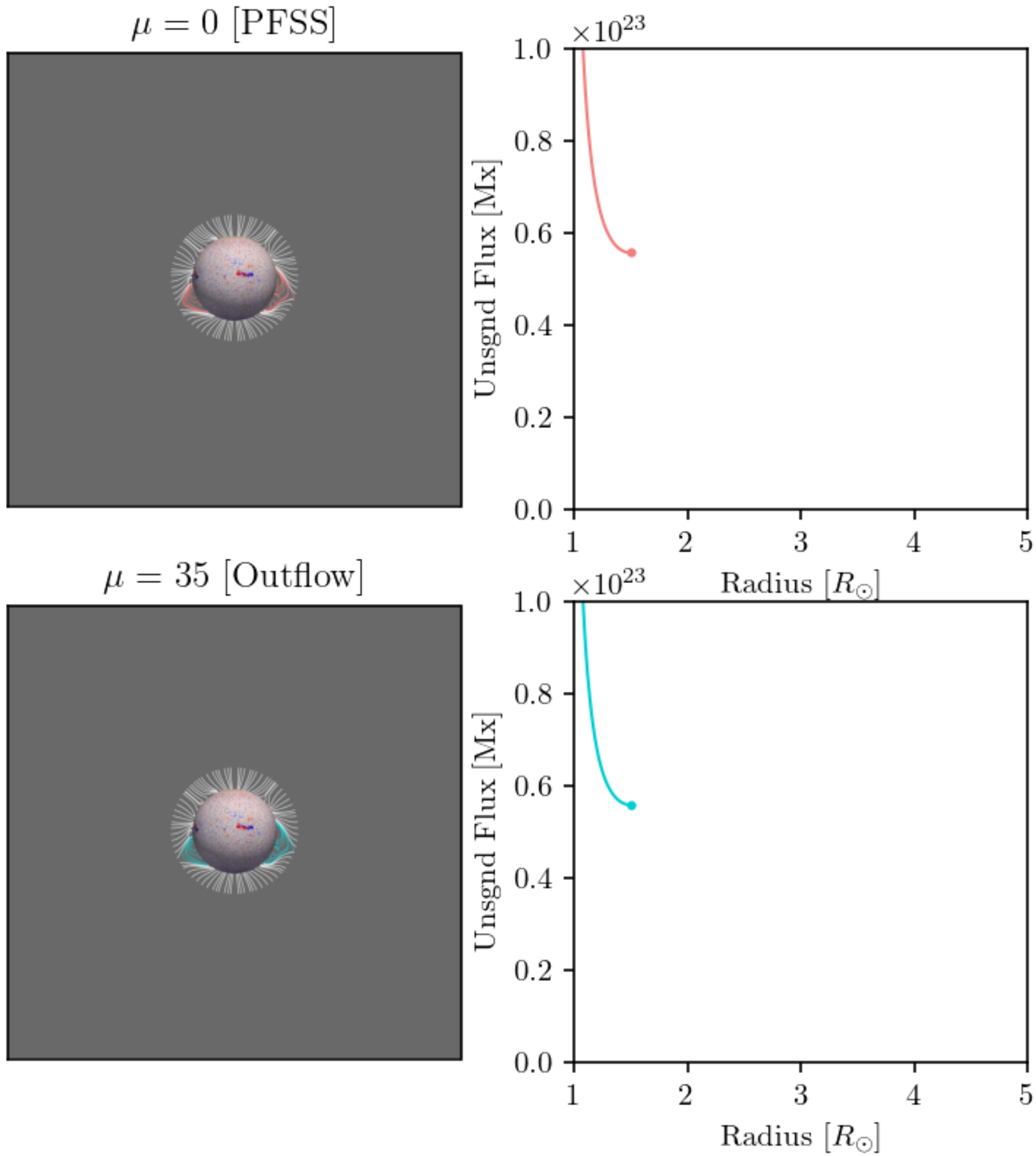

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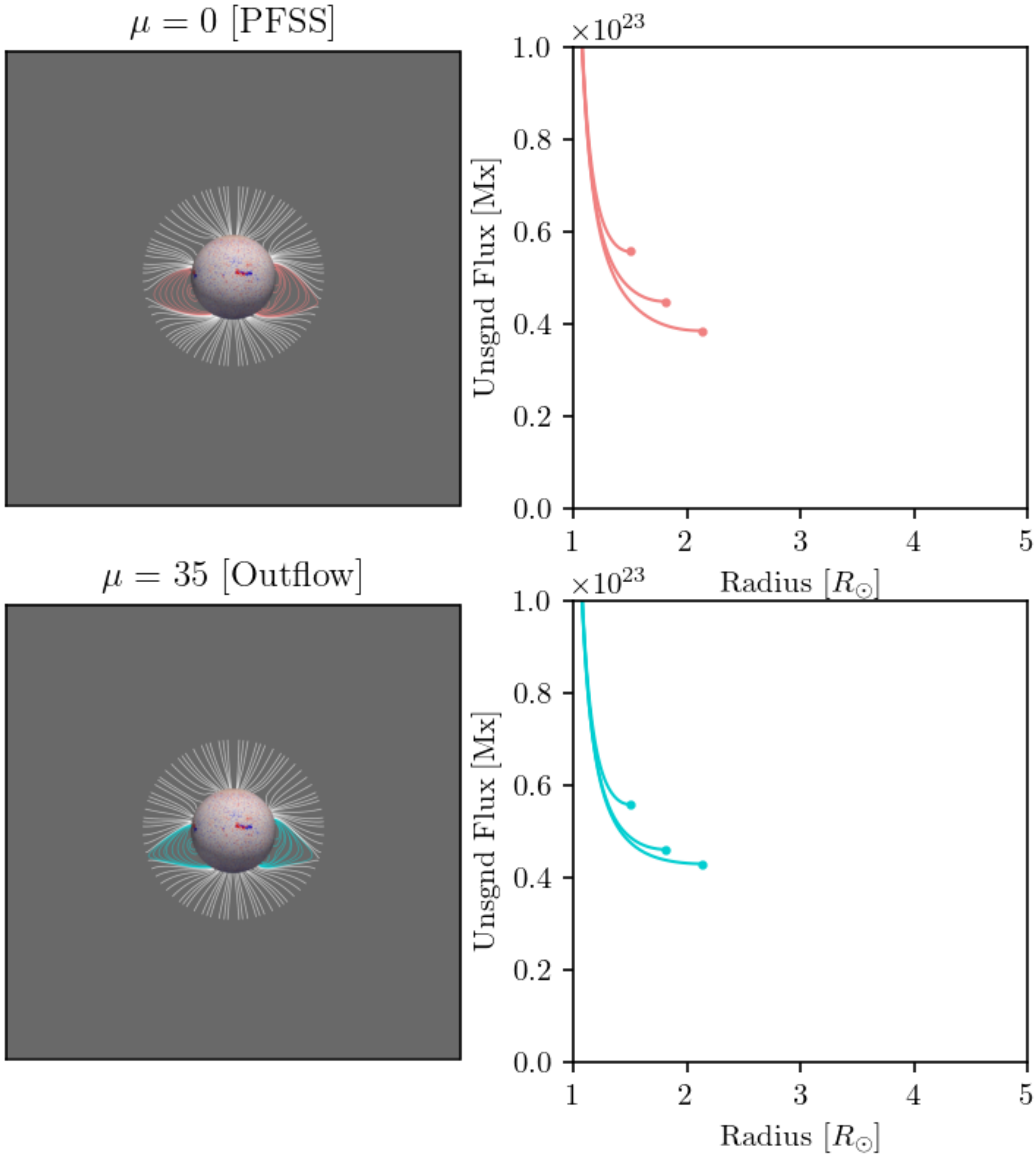

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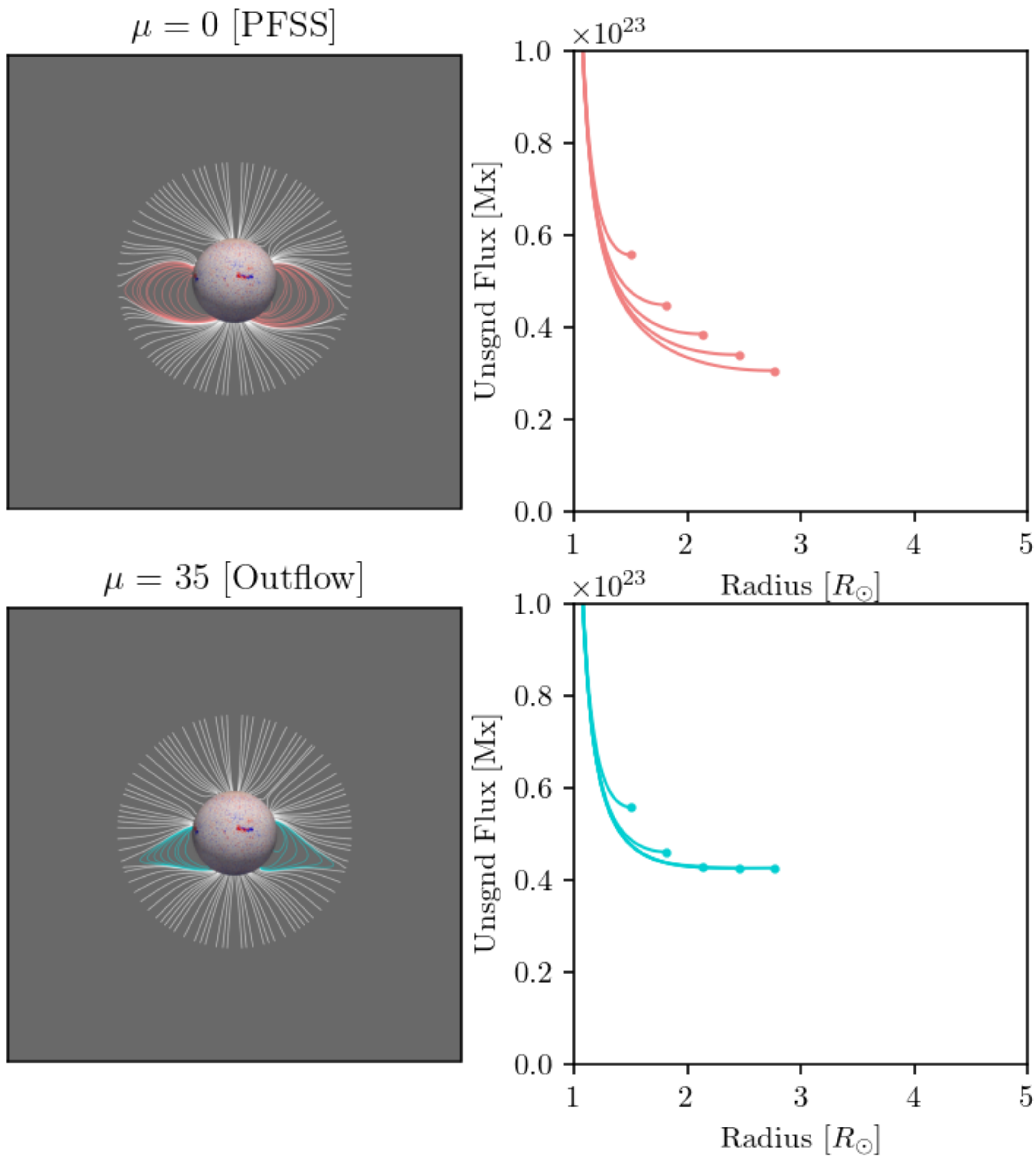

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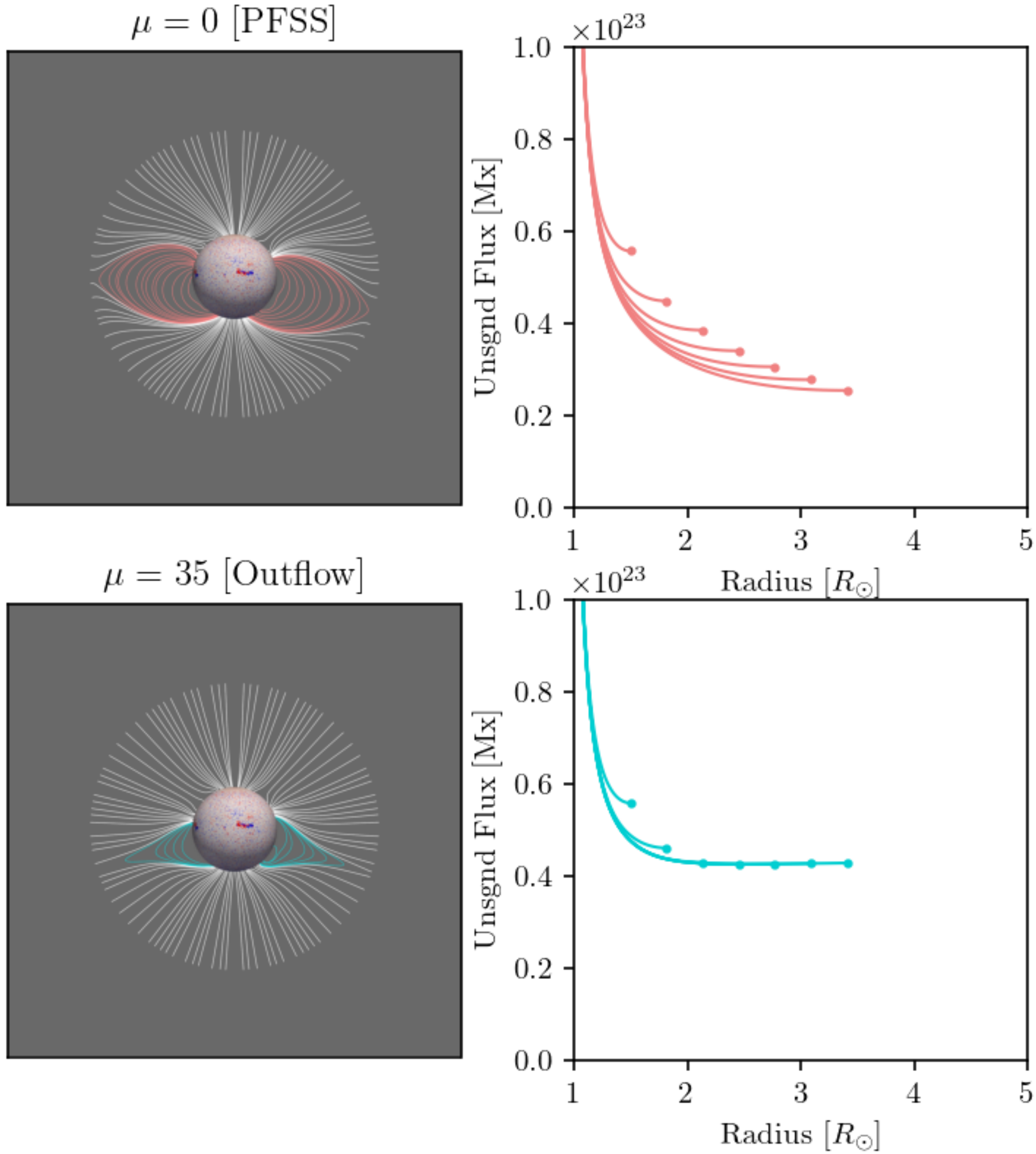

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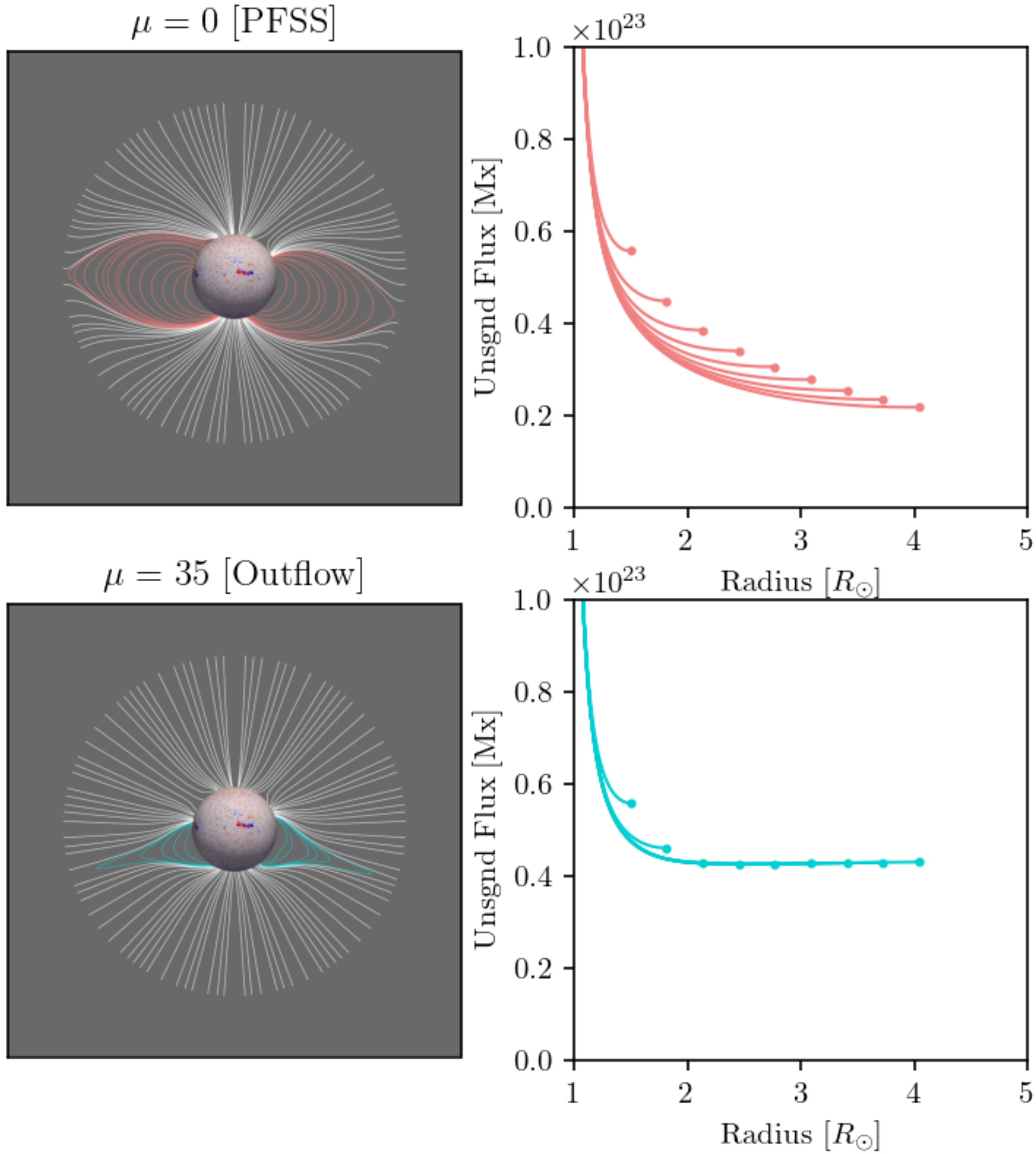

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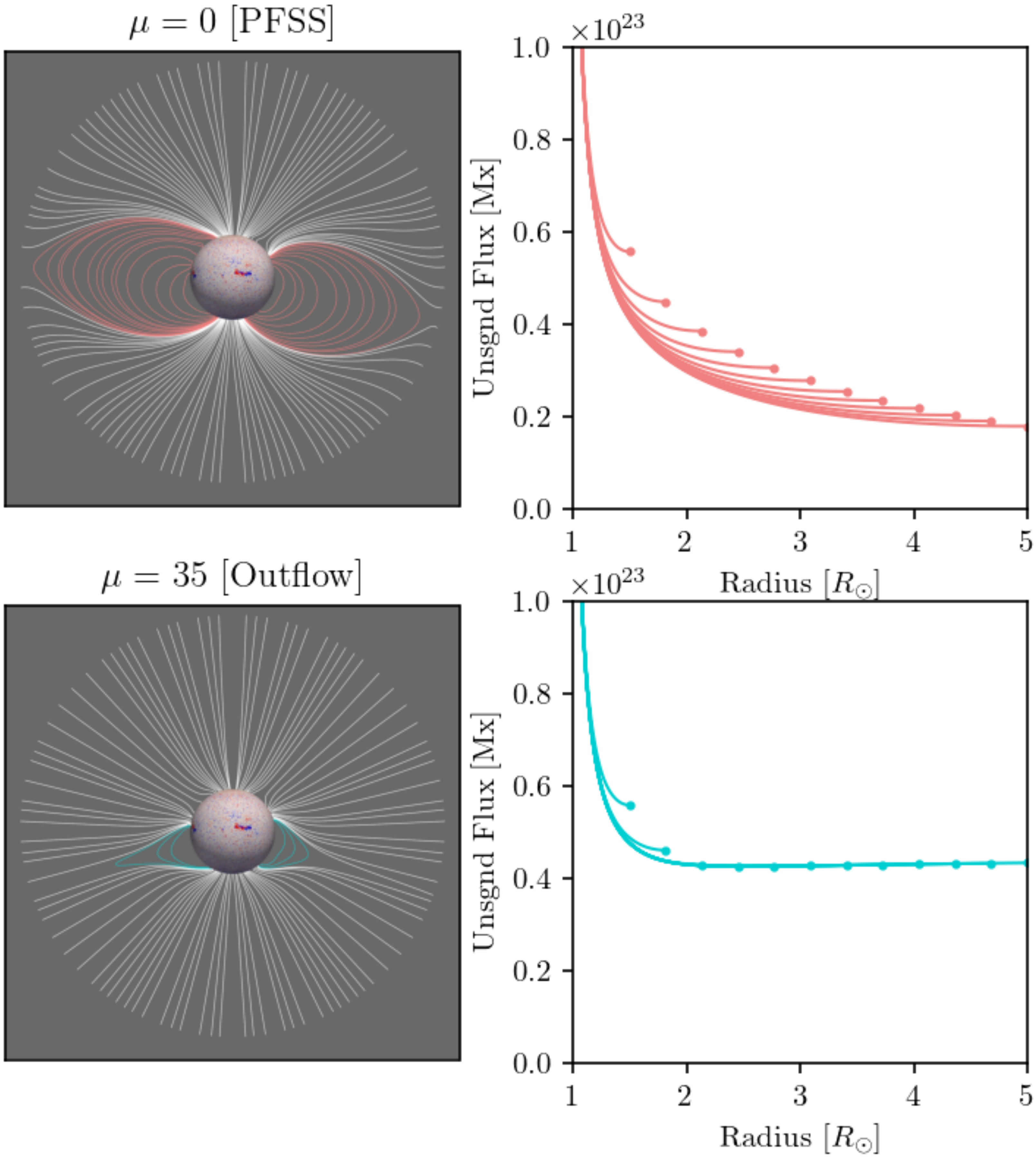

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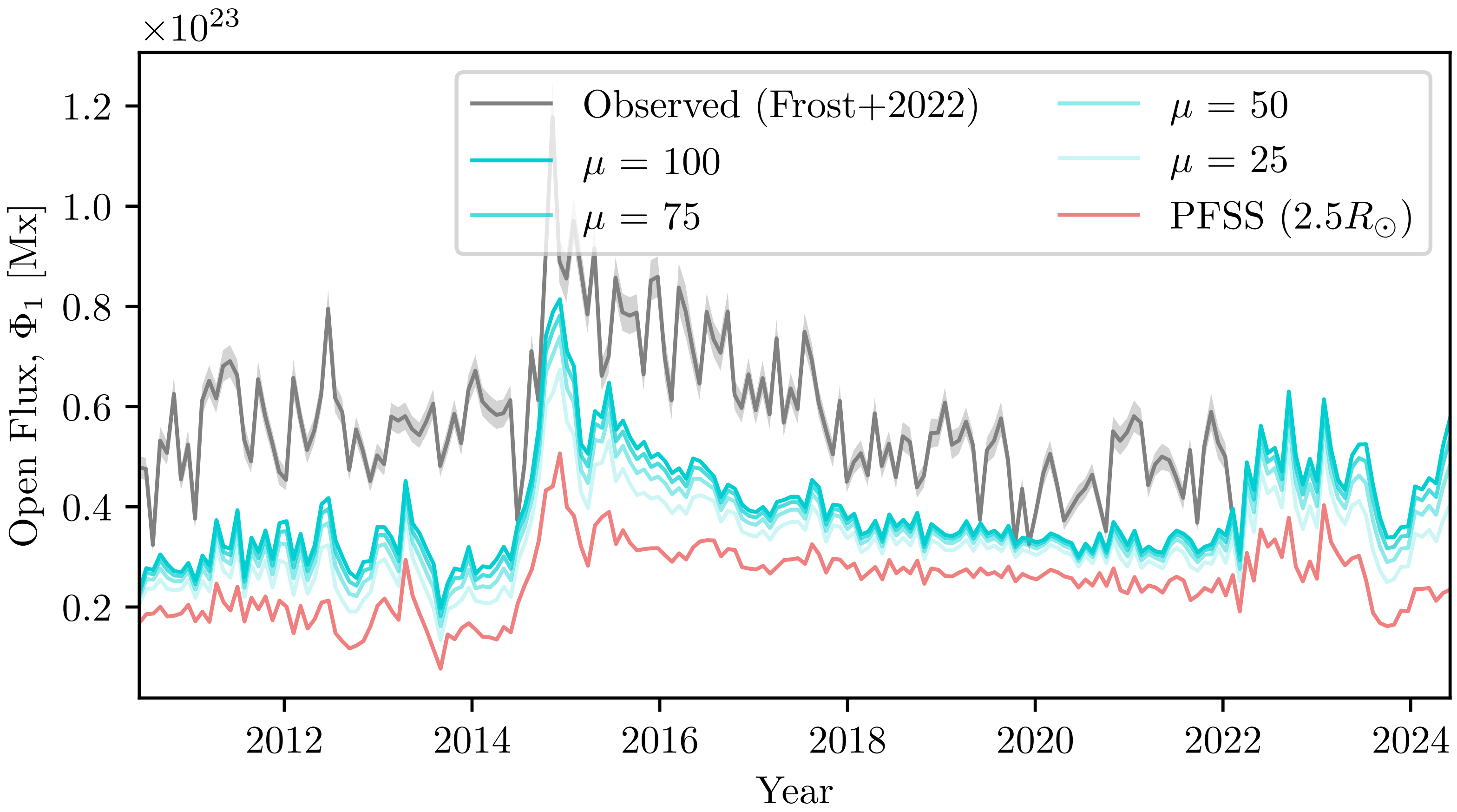

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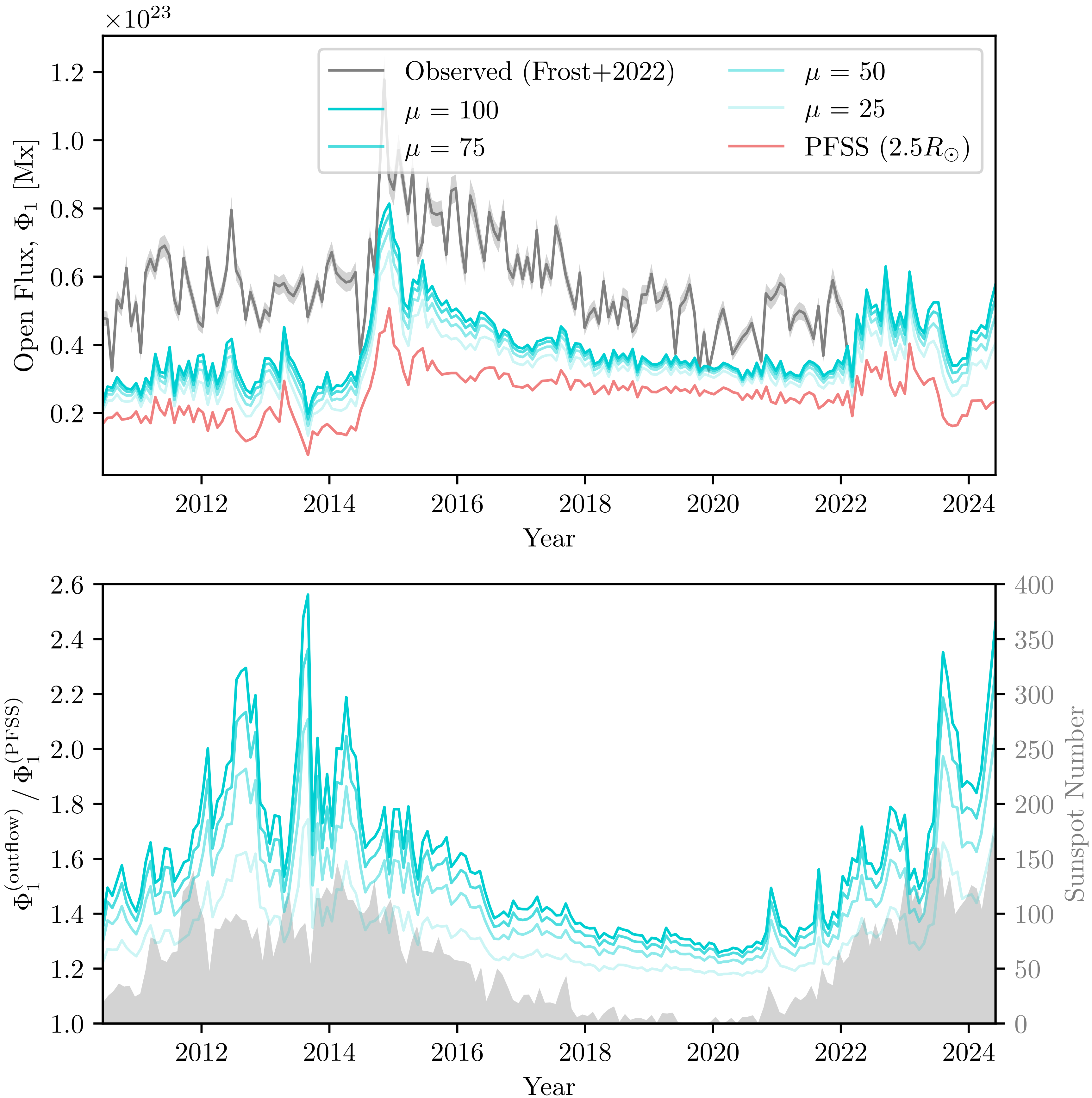

Mathematical details…

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- Depends on the dimensionless "Reynolds number" $\mu = R_\odot \nu_0 v_{\mathrm{out}}(r_0)$