Revisiting Taylor relaxation

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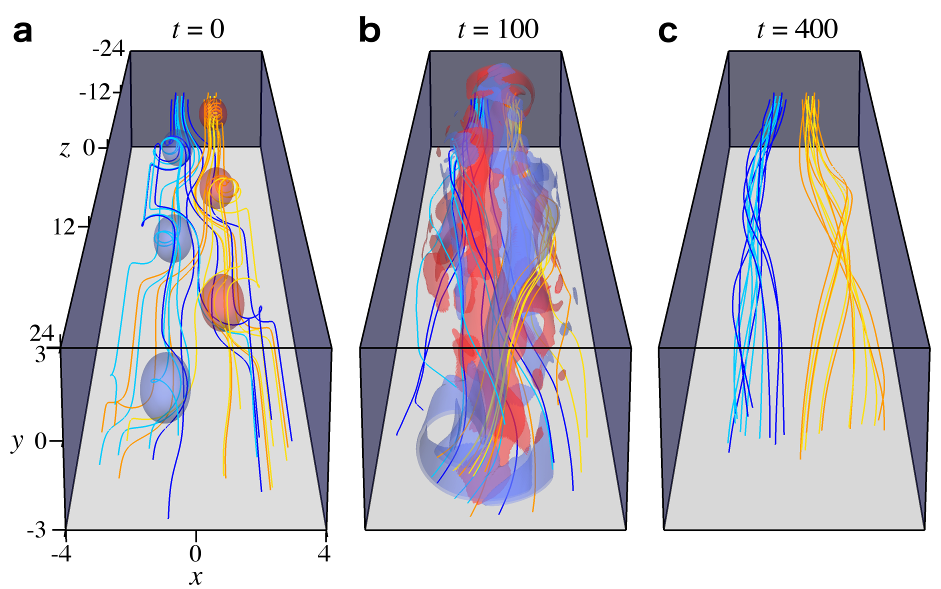
Abstract. Plasma relaxation in the presence of an initially braided magnetic field can lead to self-organization into relaxed states that retain non-trivial magnetic structure. These relaxed states may be in conflict with the linear force-free fields predicted by Taylor theory and remain to be fully understood. Here, we study **how the individual field line helicities evolve during such a relaxation**, and show that they provide new insights into the process.

Methodology - numerical simulations

Our initial magnetic field has a **complex braided structure** (the "Dundee braid" [1]). Initially there is no velocity, plasma beta = 0.01.



This initial field is evolved according to the **resistive MHD** equations (with no further driving), solved with Lare3D [2].

Lundquist numbers S=2,500 to 20,000 (maximum numerical resolution 960x960x720). Higher than our previous simulations.

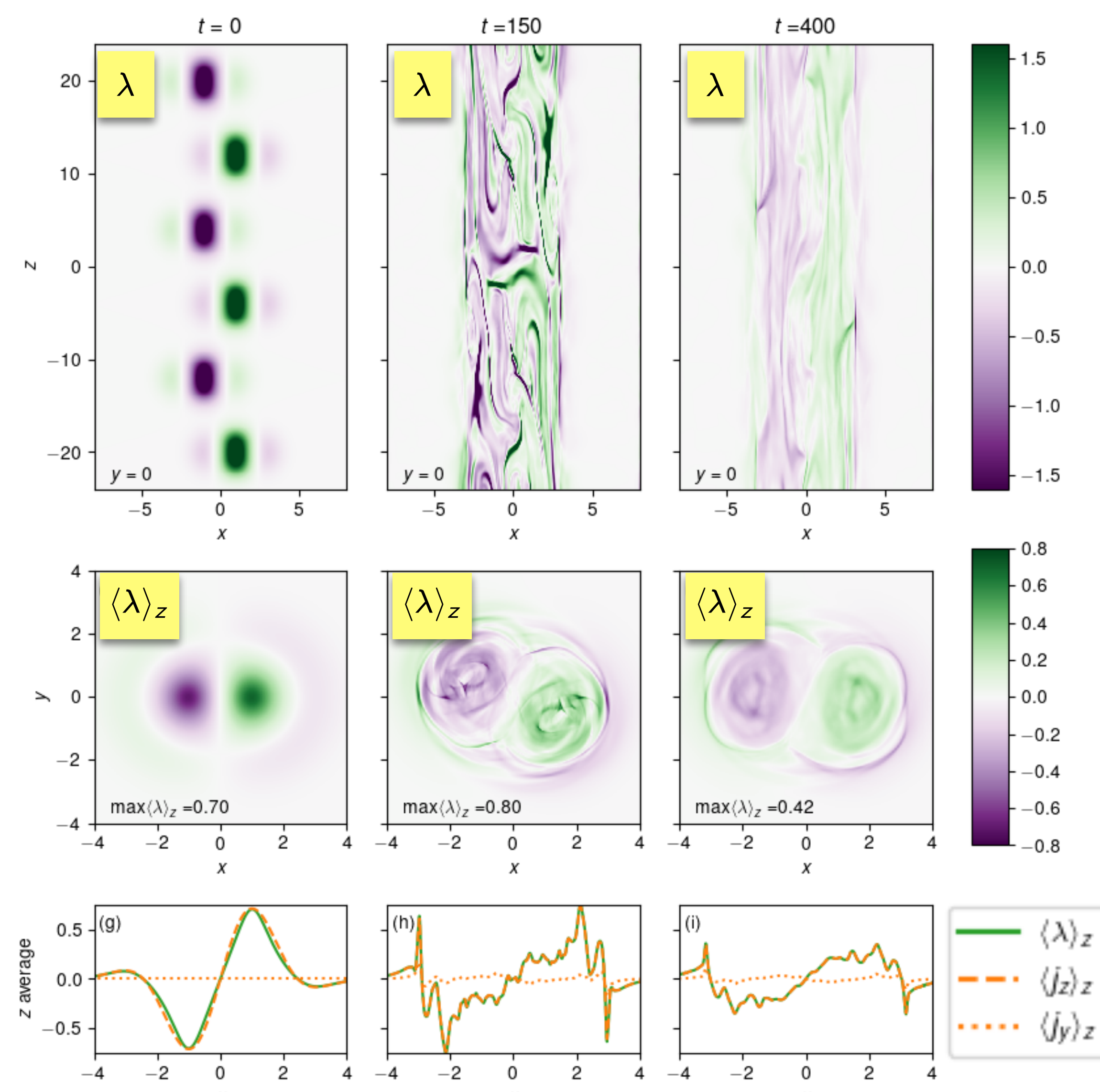
Because the initial state is not force-free, the twist regions launch torsional Alfvén waves, generating turbulence with thin current sheets and reconnection. Eventually it relaxes to a steady state subject only to slow ohmic decay. **We are interested in the structure of this relaxed state.**

1. Do we see Taylor relaxation?

The classical Taylor theory [3] is to assume that total magnetic helicity is the only invariant, implying a linear force-free final state,

$$\nabla \times \mathbf{B} = \lambda_0 \mathbf{B}.$$

To test this, we plot (below) the quantity $\lambda = \dfrac{\mathbf{j}\cdot\mathbf{B}}{|\mathbf{B}|^2}$.



The relaxed state (right) has two tubes of opposite twist, with some tendency towards uniform λ within each tube.

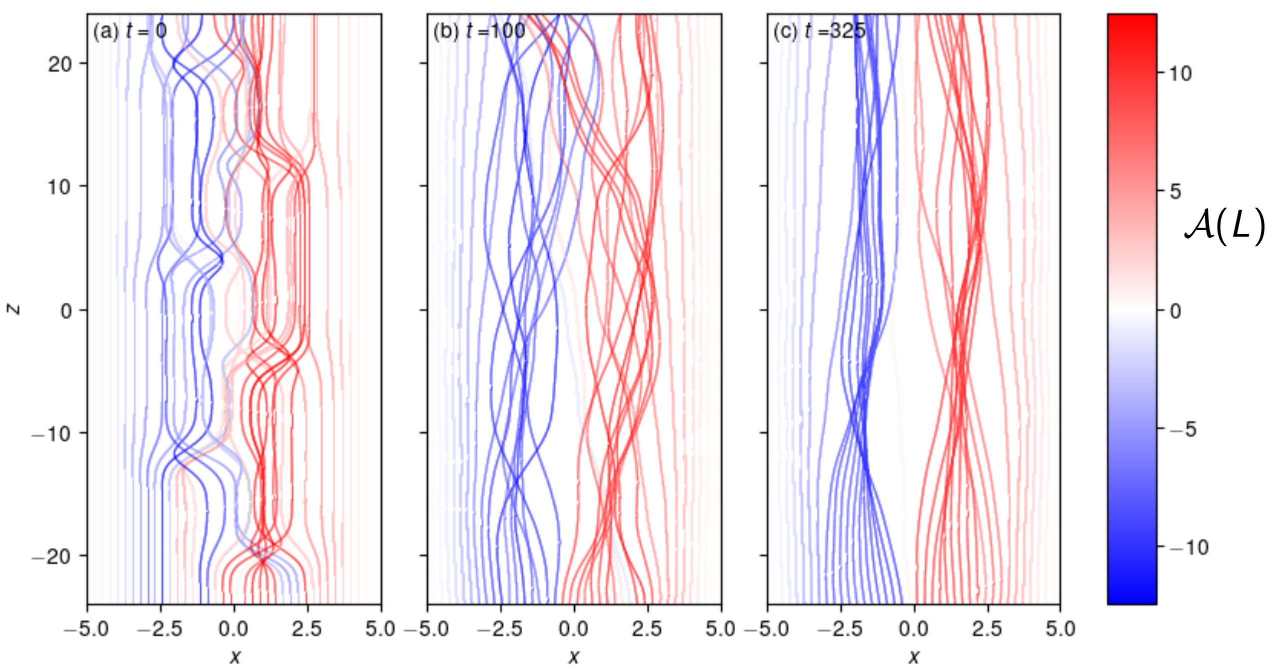
2. Persistence of field line helicity

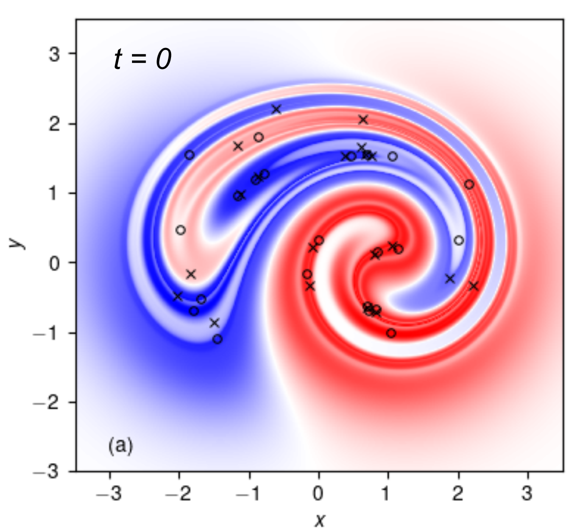
For each field line, Taylor knew that the **field line helicity**,

$$\mathcal{A}(L) = \lim_{\epsilon \to 0} \frac{\int_{V_\epsilon(L)} \mathbf{A} \cdot \mathbf{B} \, dV}{\Phi(V_\epsilon(L))} = \int_L \mathbf{A} \cdot d\mathbf{l}$$

would be an ideal invariant (the helicity per field line). But he believed it uninteresting for resistive relaxation because individual values could be changed by reconnection.

However, for highly conducting plasmas, the values are primarily redistributed rather than destroyed [4]. Our simulations show this:





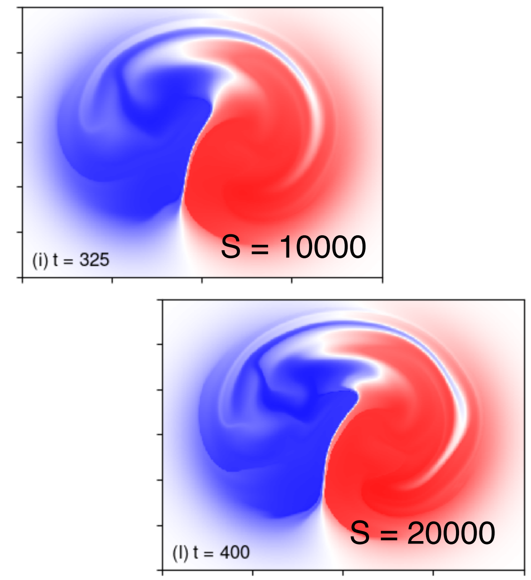
Because the reconnection is localised in x and y, the topology of $\mathcal{A}$ in cross section is constrained.

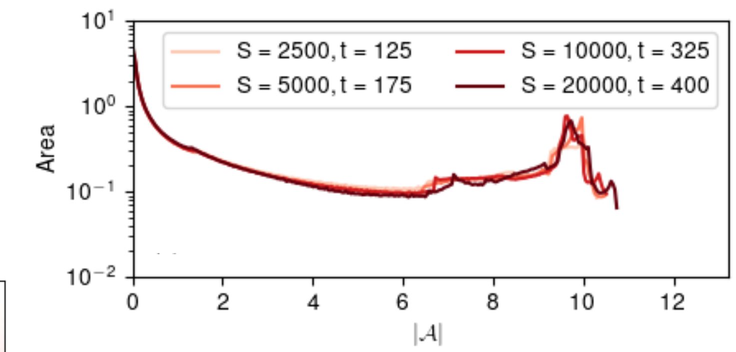
Critical points can only annihilate in ways that preserve the total Poincaré index (where maxima/minima count +1 and saddles -1).

In our initial state, the total index is +2, explaining the persistence of two flux tubes in the relaxed state.

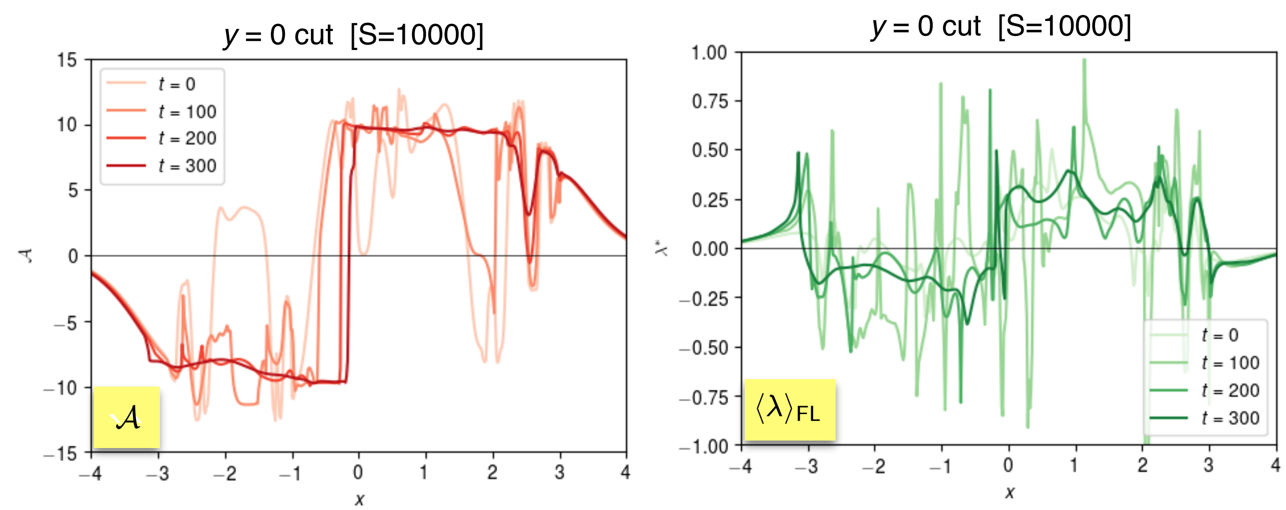
3. Uniformization of field line helicity

The final $\mathcal{A}$ pattern seems to converge with increasing Lundquist number:





Whilst both quantities would be uniform in a linear force-free tube with this helicity, we find $\mathcal{A}$ is much more uniform within each of the flux tubes than λ.



This is because field line helicity is a topological quantity that is less sensitive to local fluctuations.

References

[1] Pontin et al., Plasma Phys Contr Fusion 58, 054008 (2016).
[2] https://warwick.ac.uk/fac/sci/physics/research/cfsa/people/tda/larexd/
[3] Taylor, Rev Mod Phys 58, 741 (1986).
[4] Russell et al., Phys Plasmas 22, 032106 (2015).

The work described here has been submitted for publication…