Groups in Geometries

Geometry	Group G	Generators of G	G preserves...	Transitivity*	Uniqueness**	Classification***	Fixpoints
$\mathbb{E}^2$	$\mathbf{x} \mapsto A\mathbf{x} + \mathbf{b}$ $A \in O(2,\mathbb{R})$	reflections	distance angles	on flags	3 non-collinear pts	reflection rotation translation glide reflection	line 1 point - -
S^2	$O(3,\mathbb{R})$	reflections	distance angles	on flags	3 non-collinear pts	reflection rotation glide reflection	line 2 (antipodal) points -
Aff	$\mathbf{x} \mapsto A\mathbf{x} + \mathbf{b}$ $A \in GL(2,\mathbb{R})$	$Isom(E^2)$ and $\begin{pmatrix} \alpha & 0 \\ 0 & 1 \end{pmatrix}, \alpha \in \mathbb{R}\setminus\{0\}$	collinearity $\Rightarrow$ parallelism ratios of lengths on a line concurrence of lines ratios of areas	on triangles	3 non-collinear pts		
$\mathbb{RP}^1$	$PGL(2,\mathbb{R})$	projections of lines to lines	cross-ratio	on triples of points	3 points	$\frac{ax+b}{cx+d}$, $a,b,c,d \in \mathbb{R}$ $ad - bc \neq 0$	
$\mathbb{RP}^2$	$PGL(3,\mathbb{R})$	projections of planes to planes	cross-ratio of 4 collinear points	on quadrilaterals (4pts, no 3 collinear)	4 points (no 3 on a line)		
$M\ddot{o}b$	$PGL(2,\mathbb{C})$	az, $z+1$, $1/z$ ($a \in \mathbb{C}$)	cross-ratio angles	on triples of pts	3 points	parabolic, conj. to $z+1$ non-parabolic, conj. to az elliptic $\lvert a \rvert = 1$ hyperbolic $\lvert a \rvert \neq 0$, $a \in \mathbb{R}$ loxodromic $\lvert a \rvert \neq 1$, $a \notin \mathbb{R}$	1 point 2 points no atractors/repellers attractor & repeller attractor & repeller
$\mathbb{H}^2$	$G^+ = PGL(2,\mathbb{R})$	reflections	distance angles	on flags on ideal triangles	3 non-collinear pts 3 pts on absolute	reflection rotation parabolic translation hyperbolic translation glide reflection	line 1 point 1 point on absolute 2 points on absolute 2 points on absolute

*Transitivity = "G acts transitively on ..."
**Uniqueness = "$g \in G$ is uniquely determined by the images of ..."
***Classification = "types of elements of G"

$PGL(n,k) = GL(n,k)/\pm I$
n = dimension, $k = \mathbb{R}, \mathbb{C}$

G^+ = or.preserving subgroup of G