Cultural Evolution of Material Knot Diversity

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Declaration

The work in this thesis is based on research carried out at the SPOCK Group, the Department of Mathematical Sciences, England. No part of this thesis has been submitted elsewhere for any other degree or qualification and it is all my own work unless referenced to the contrary in the text.

Contents

Chapter 1

Introduction

Knots are an important part of our everyday life. From our shoelaces to securing loads on a truck, knots make an essential tool and are tied by many of us everyday. They are used the world over, by almost every society. The Human Relations Area Files ethnographic database [1] contains 1,900 references to knotting across 228 different cultures. It is not just humans who use knots, knots have been found in the animal kingdom as well. Certain gorillas tie knots in their nests, granny knots have been tied out of saplings and creepers as well as the slightly more complex reef knot [2]. The Ploceidae or Weaver Bird builds its nest out of knots, weaving an intricate pattern to attract a mate [3]. Knots are part of both human and animal life, woven into important rituals and everyday practices.

There has been archaeological evidence which suggests that knots played a crucial role in the development of early humans, along with the use of fire or the wheel. It is difficult to say when the first knot was used, as knots are usually made of perishable material and so are subject to decay, but artefacts which probably require knots have been found to date as far back as 300,000 years ago [4]. Material knots have been found in sites where bodies and artefacts have been preserved in conditions with sub-zero temperatures, a completely dry environment or one which prevents decomposition [5]. Knots were found as part of the "Ice Man's" equipment when he was discovered in 1991 south of the Italian-Austrian border. His body and equipment had been frozen solid and preserved for over 5,400 years. The knots he had in his possession were identified as single hitches, overhand and half knots, reef knots and overhand bends amongst others. These knots formed part of the "Ice Man's" net, bow and clothing. Other preserved knots have been found in various countries in bogs, used as nooses [5], textiles and fishing lines, dating as far back as 3500 BC. Knots have been observed from Ancient Egypt in both Archaeological remains and texts [6], the earliest dating to 1350 BC and found in Middle Egypt.

Knots play a large part in mythology, an example being the legend of the Gordian Knot [7]. The legend says that a poor peasant called Gordius arrived into Phrygia in an ox cart. Unbeknown to him, it had been foretold that the future king would arrive in this way and so he was crowned. In gratitude, Gordius dedicated the cart to Zeus, tying it up with a knot, named the Gordian Knot. It was then foretold that the next king would be the one who could unravel this knot. Many tried, but the knot was too complicated. In 33 BC Alexander the Great came along and cut the knot with his sword, a solution that seemed to go against the spirit of the challenge. The phrase "Gordian Knot" is now used to refer to a complicated problem.
These findings demonstrate that knots are an important part of human's material culture and an integral part of human history and development.

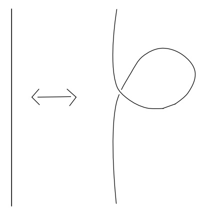
Given knots are something we use daily, have we ever stopped to wonder why we use the particular knot we do for a given purpose? For example, a lot of us regularly tie our shoelaces, going through the motions and not really thinking about the knot we are tying. Why do we tie our shoelaces in this way? Is it the optimal knot for the purpose or do we just blindly follow the algorithm we are taught as children about bunny ears and a hole?

Maybe studying the knots we use needs to be viewed from a different point of view. Maybe the reason we use knots can be studied using a mathematical analysis?

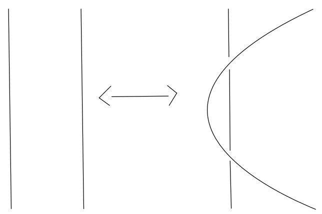
The study of knots by Mathematicians really took off in the 1800s. Knots had been looked at from a topological point of view before this time but in the 1860s Lord Kelvin hypothesised that atoms were "knots of swirling vortices in the aether" and so chemical elements would correspond to knots. This led to Peter Guthrie Tait's work into attempting to characterise all unique knots, believing a table of elements could be designed in this way. Unfortunately, there is not a one to one correspondence between elements and knots but a desire to classify knots was born.

Mathematicians view knots as a closed curve, considering the ends of a knot tied in string to be glued together, so the knot cannot be undone. Knots are classified by their minimal crossing number and denoted by this in knot tables, listing all distinct knots. However, a knot can be drawn in one way, and a string moved but the knot not really changed, resulting in a projection of that knot that looks different. The difficulty then is determining when two knot drawings or projections actually describe the same knot.

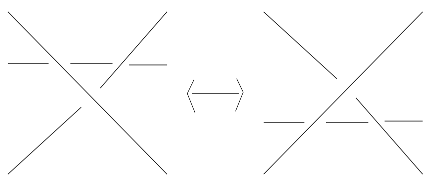
In 1926 Kurt Reidemeister showed that two knot projections are the same if and only if they can be deformed into each other by a sequence of Reidemeister moves [8]. These moves allow us to add or remove a twist from a knot strand, pull one strand completely over or under another and pull one strand completely over or under a crossing.



Reidemeister I move: We can add or remove a twist from a knot strand.



Reidemeister II move: We can pull a strand completely over or under another.



Reidemeister III move: We can pull a strand completely over or under a crossing.

From this theorem, the term *Knot Invariant* can be defined. A Knot Invariant is something we can calculate for one knot that is unchanged or invariant under Reidemeister moves. This means if we have two different projections of the same knot then the Knot Invariant calculated from either projection will be the same.

Using these Knot Invariants, Knot Tables detailing all distinct knots can be created with certainty and so we can see how many knots there are and study their differences. This leads to the question of which knots used in material culture relate to which mathematical knots from these tables. Once we know which knots in material culture relate to which mathematical knots we may ask why these particular knots are used.

Culture is an important concept in the study of human behaviour. There are many definitions of culture. Boyd and Richerson define culture as "information capable of affecting individual's behaviour that they acquire from other members of their species" [9]. This does not include information acquired genetically or learned individually. Cultural evolution can be described as a process of "descent with modification" [10] where socially learned behaviour is passed within a population. This socially learned behaviour is referred to as a trait. Each individual may learn a trait then pass it on, adding their own modification as they go. In this way we get a rapid evolution, often much quicker than genetic evolution. A trait could also be misremembered or copied incorrectly, resulting in a mutation. These mutations may later on be inherited and passed to others in the population, in the same way as the original trait.

This means that human behaviour is not completely determined by inherited information, but also by socially inherited information. Evolution of culture is treated in a way analogous to the inheritance of genes, but the transmission of cultural traits can be passed in ways other than from parent to offspring, referred to as vertical transmission. Cultural traits can be transmitted horizontally, from those of the same generation or obliquely, from others of the parent's generation to the offspring [11]. Traits can also be transmitted via private teaching in a one-to-one way, or through mass teaching and observation in a one-to-many scenario, such as in classrooms or via the media.

In such a way the skill of knot tying can be transmitted within a population. Knots could be being used purely because that is the first knot we are taught as children by our parents, or we could be transmitting the information horizontally or obliquely using a much wider range of information to decide which knot to use.

Using mathematical Knot Theory and a study of knots in material culture, I aim to answer the following questions;
Why are there so many knots, why not more or less?
Are these knots needlessly complicated or optimised for their purpose?
What are the common features in knot design and how are these preserved?
How many of the possible knots are utilised and what is the reason for this?
How does the learning environment effect the fidelity of knot transmission?

Chapter 2

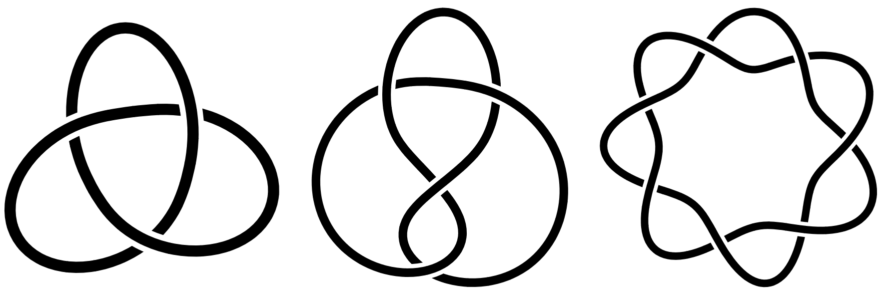
Literature Review

Knots have been of great interest to scientists for many years. They have been seen in the tangling of molecules [12], studied in DNA and constructed in knotted light. In our everyday life we encounter knots regularly, tied in our shoelaces, wrapped around parcels, for decorative purposes and as fashion accessories.

Many people have a keen interest in knot tying, for example sailors and climbers but other professions need to know and use knots regularly too. An attempt has been made by Ashley among others to collect and create an encyclopaedia of knots and their uses [13]. From Ashley's book of knots we can gain a great insight into the sheer volume and wide range of knots used and discover some of the history behind how and why these knots are used. Ashley's book contains over 3,800 knots and whilst he goes to a great length to provide as much information as he can for each knot, some knots are repeated and some knots don't have a lot of information about them. However, from Ashley's work we can put together a picture of the quantity of knots used, over a range of uses, and get an idea of which are best for these uses.

Pairing Ashley's work with other studies may provide us with more information about knot usage. Studies into knot strength and suitability have been carried out in studies comparing different types of rope and different knots tied, a factor extremely important to those who use knots for purposes such as climbing. It is known that when a knot is tied in a piece of rope it weakens the rope so it is important to choose your knot and rope carefully. Ropes used for climbing certified under the UIAA (International Climbing and Mountaineering Federation) [14] guidelines, which means

they have been tested to ensure they can withstand the expected maximum fall and impact felt by a climber.
Pieranski et al. [15] study the strength of knots by localisation of the breaking point of knots when under strain. In order to easily pinpoint the location of the breaking point cooked spaghetti was knotted and then put under strain by being pulled gently by hand. These tests were recorded by a digital camera with high recording speed so the video could be viewed later and the knot breakage determined. The knots in this experiment were denoted by their notation in the Rolfsen Knot Table [16]. In Pieranski et al.'s study it was found that the weakest knot of all was the Overhand knot (knot 3_1). It was also noted that knot strength increased as the crossing number of the knot increased, which is what we may expect. The exceptions to this rule were the knot 7_1, which was worse than all knots of 6 crossings, and the Figure-Eight Knot, (knot 4_1) which was stronger than all knots of 5 and 6 crossings, and knot 7_1. It was found that the knot breakage did not occur in the internal region of the knot, breakage was close to the entry to the knot. These studies give us an idea of the suitability of knots for certain purposes and leads us to question why the Overhand knot (3_1) is so widely used for a range of purposes when it is shown to be the weakest knot of all.



Knots 3_1, 4_1 and 7_1

In addition to the strength of climbing knots, the vast range of possibilities of tie knots has been studied. Fink and Mao [17] attempt to predict all aesthetic tie knots by modelling their construction through random walks. They define a tie knot by a sequence of moves describing the wrapping of the tie by the orientation and location of the tucks used to tie the tie. These moves can be represented as walks on a triangular lattice and so the amount of possible tie knots can be calculated. Fink and Mao demonstrate that there are 85 possible sequences and so 85 possible distinct ways to tie a tie. It is interesting to note that whilst there are 85 possibilities, only four of these are commonly used as ways to tie a neck tie. Whilst Fink and Mao only considered ties tied with the wide end of the tie, Hirsch et al. [18] consider extend the tie knot possibilities by including ties tied with the thin end. This takes the number of possible neck ties up to a staggering 177,147. One thing is clear from these studies, the amount of possible tie knots is huge, but only a small percentage are observed in real life, leading us to question the reasons behind this.

The range of evidence in Ashley and that gathered through studies suggests there is a huge range of diversity in knots, but these studies do not suggest why. An answer to this may lie in the way we learn.

The weaving of knotted textiles in Iran was studied by Tehrani and Collard [19]. Interviews were conducted to try to determine how weaving was taught. The results from these interviews reported that most young girls were taught to weave by their mothers and that the teaching of weaving techniques is mainly through a mixture of demonstration, participation and intervention, with little verbal instruction. The techniques learned in the weaving process are similar to the techniques needed for knotting and so we may expect knots to be transmitted in a similar way, with high learning from parents and learning from others once skilled. We may also expect the teaching techniques to be similar, through demonstration, participation and intervention.

As well as interviews, transmission chain experiments are often used to explore the effect of teaching techniques on a sample of the population. The behaviour which is observed in these experiments may be indicative of the population as a whole. Linear transmission chains operate through a "Chinese Whispers" method. Information is passed through a chain of participants where each participant learns the information, attempts to recall it, and then passes it to the next participant in the chain. The changes that occur in the chain can be measured and give an indicator of the degradation of information in the wider population [20]. Different samples can be manipulated to more accurately model the population or hypothesis which is being tested. Multiple chains can be ran at the same time with different instructions to study the effect of instruction or members of chains can be replaced to model the introduction of new members to a population. These chains are useful for studying how information is best passed within a population, but they may also help understand why we learn certain things.

Social learning is subject to a range of selection pressures or biases which can affect the likelihood of a trait being preserved. These social learning pressures can be categorised into content and context biases [21].

Content biases are based on the attributes of the trait being transmitted. These attributes can have an effect on the probability of the trait being spread within the population. An example of a content bias may be preferring to use a steel axe over a stone axe as the steel axe requires less effort to cut down a tree [21]. Other examples include farmers switching to a different seed because it produces a better yield or a manufacturer choosing to sell a different product based on monetary return. These attributes are based on the perceived costs and benefits of traits, affecting the likelihood of transmission. However it could be quite costly to an individual to adopt traits in this way as they may have to waste time trying all available options to see which is most effective. It may also not be apparent straight away that the new trait is preferential to the old one [10].

Context biases refer to the environment in which the trait appears. Context biases include model-based biases and frequency-dependent biases.

Model-based biases arise from the likelihood of a trait being transmitted increasing depending on the individual who is observed displaying this trait. An example of this would be the behaviour of a celebrity being replicated by their fans, such as buying the same brand of clothing as the celebrity. In the same way a trait could not be transmitted because it is one associated with a group which are not prestigious or whose beliefs differ to those of the observer.

Frequency-dependent biases arise from the likelihood of a trait being transmitted increasing depending on the rate of occurrence of its observation. For example, people may be more likely to buy a new smartphone as it is the one everyone else may have and they wish to have the same technology as they observe everyone else having. This is referred to as conformity bias. In the same way a trait could not be transmitted because it is seen to be too common, users may not buy the latest smartphone as they do not with to conform to the same technology as everyone else, this would cause a bias towards rarer behaviour or traits. Sometimes opting for the most common behaviour is less costly to the individual than trying all the alternatives. However, simply opting for the most common trait could cause traits which may not be the most effective, or out of date to be retained in the population [22].

These biases may give us an idea to the likelihood of certain behaviour being preserved in a population and may suggest the likelihood of certain knots being used.

Knot tying has been used as a tool by Muthukrishna et al. in experiments to test the effect of multiple models on learning [23]. As knot tying only requires a piece of rope to do this makes it an accessible tool for which to experiment with. In this study the group of participant were asked to tie a series of knots commonly used by rock climbers. The study ran through two chains, each with ten generations. In the both chains participants would learn how to tie the knots from the generation before them. In the first chain participants were only allowed to learn from one person in the generation before them. In the second, participants could learn from five models in the generation before. The first generation in both chains were trained by the experimenter to become "experts" at tying the knots. Other generations created an instructional video for the tying of the system of knots by a camera strapped to their head. The next generation would then be given this video along with a score which measured how well the participant tied the knot series. This score was measured on a scale used when assessing sutures when training surgeons and was judged by human raters [24]. The results showed that knot tying skills declined throughout all generations but declined more slowly in those in the five-model chain than the one-model. One of the issues with the experiment was that the participants in the five-model chain did not have time to view all of the instructional videos presented to them. Another issue was the way the knots were judged. The knots were given a score based on a set of requirements observed by a rater, but the knots were not studied to determine whether they were mathematically the same. However, the way this study was set up gives us a good idea of the way in which to approach knot transmission chain experiments and that the sample size of demonstrators may affect the fidelity of transmission.

In order to explore the difference between individual and social learning Derex et al. [25] ran a virtual experiment concerning net building and fishing. Participants were required to construct a net on a square grid using a limited amount of rope of various thickness and knots of various size. Nets were tested and given a score based on how many fish the simulated net caught. During each of the fifteen trials, participants could view their previous net and score. The participants were placed into different groups under three different treatments, participants were unaware of who was in their group and which treatment they were in. In the individual learning treatment, participants could see the last trial and cumulative score of the rest of their group members. In the product copying treatment, participants could see the different scores of each of their group members and the corresponding nets. In the process copying treatment , participants could see the different scores of each of their group members, the corresponding nets and the step-by-step information for building that net. Participants had 30 seconds in the individual learning treatment to view the information and 90 seconds for the other two treatments. The nets were scanned pixel by pixel for similarity and scored. The process similarity was judged by viewing the net building actions as characters in a string and so the similarity of the string was measured. Scores for net building improved throughout all treatments and younger participants generally performed better than other participants. The difference between performance in the individual and product learning treatments was not significant but the process copying treatment demonstrated a significant advantage. This indicates the importance of social learning of the knotting process in this virtual net building task which may also be the case for real net building, however the results could have been skewed by the fact that the task was virtual and the observation that the age of particiapnts made a difference on performance. We may expect social learning mechanisms to be also important for knot learning as nets are made up of a system of knots.

Pairing knot studies with an assessment of the knot learning environment, we attempt to answer our research questions using the methods described in the next section.

Chapter 3

Methods

To answer our research questions, we will be creating a knot database and using experiments which will be discussed in this section.

3.1 Knot Database Analysis

I am making a database of knots used commonly, starting with those in the Ashley Book of Knots [13]. The layout of this database an and explanation of the fields is given below.

ABOK No.	The knot number it first appears as in Ashley
Also appears as	Any other numbers the knot appears as in Ashley
Knot Name	Name as given in Ashley
Crossing number ABOK picture	Crossing number as given in Ashley
Mirrored	Whether the knot is mirrored based on Conway notation (only applicable for some knots)
Knot	Common name used by Mathematicians (if there is one)
Prime	Whether or not the knot is prime
Knotplot input	Knotplot input if known (based on Conway notation)
Knot Atlas notation	Knot Atlas notation if known (for larger knots Knotscape notation is used)
Crossing number	Reduced diagram crossing number
Link	Whether or not knot is a link
Number of components	If link how many components it is made of
Linking no	Linking number if knot is a link
Notes	Any notes relating to considering knot as joined ends
Related knots	Any related knots mentioned by Ashley
Uses	Uses given by Ashley
Use comments	Any comments on useage
ABOK classification	Ashley's classification, Important, Strong, Practical etc.
Alternative names	Any alternative names given by Ashley
ABOK Image	Original image from Ashley
KnotPlot Video	Video showing deformation from Ashley's knot to a known mathematical knot

3.1.1 Method for knot reduction

In order to complete this database I use the following steps.
First I take Ashley's image of a knot and draw it out. Next I need to determine how to join up the ends to consider the knot as a closed loop. This has a few cases;

Case I: Knot is tied in one piece of rope with both ends free. In this case I can join up the ends without any additional crossings so do not have to worry about choosing how to join the ends up. The joining is somewhat natural.
Case II: Knot is tied in one piece of rope but the ends are not free. In this case I need to consider the different knots resulting from the ends joining up in different ways. As joining up the ends may create new crossings I need to keep the amount of new crossings minimal and consider all possible cases.
Case III: Knot is tied in more that one piece of rope with more than one free end. In this case I must be careful to join up the ends correctly. I must look to the knot tying instructions in Ashley to see if they give any clues about the natural way to join the ends. As the knot is tied in more that one piece of rope I must be careful to ensure that joining up the ends does knot reduce the number of components of the knot. Again if there are multiple choices I must consider all cases.
Case IV: Knot is tied in more that one piece of rope but ends are not free. Again I must ensure that joining up the ends results in a knot with the same number of components as pieces of rope. If joining the ends creates new crossings I must consider all cases.
Case V: Knot is a braid. If the knot is drawn in Ashley as a braid, with free ends at the top and bottom and over and under crossings not doubling back on themselves, I determine the braid word and consider the knot as the braid closure.

Once I have joined the ends up I draw the knot in KnotPlot [26] and see how many crossings it has. This is the number I use for the crossing number of the ABOK image. With the knot drawn with smooth tubes, I relax the knot using KnotPlot and record the screen using gtk-recordMyDesktop [27]. This program saves a video in an .ogv format which I then convert to .avi using Mencoder in Linux terminal. I convert to .avi as it is a more widely supported format that .ogv. This is used as the knot video.

Once the knot has been relaxed I compute the knot's Dowker notation [28] and HOMFLY (sometimes called HOMFLY-PT) polynomial [29], [30] using KnotPlot's inbuilt calculators. The Dowker notation describes the projection of the knot by associating an odd and even number to each crossing. A string of numbers is produced which describe precisely the knot diagram, allowing a computer to eaily read this code and draw the knot. As the Dowker notation depends on the projection of the knot, it is not a knot invariant. The HOMFLY polynomial, however, is a knot invariant. By assessing the crossings of a knot we can produce a polynomial. This polynomial is the same for all projections of the knot and can also be used to differentiate between handedness of the knot.

If the knot has less than 10 crossings I can generally identify it using KnotPlot's identify command. If the knot cannot be determined using KnotPlot's identify command I input the Dowker notation into KnotInfo's Knotfinder [31] which identifies the knot for me providing it is prime. If Knotfinder cannot determine the knot then the knot is either composite, meaning it is made up of two smaller knots, or the knot has more than 13 crossings. I next input the Dowker notation into Knotscape, this program will tell me which knot I have if it has less than 16 crossings and is prime. If the knot is not prime Knotscape will tell me. In the case where the knot is composite, I need to manually determine which knots it is made up of. If the knot is prime and larger than 16 crossings, I note down the length of the minimum Dowker notation as computed by Knotscape and leave the knot name blank as knots with more than 16 crossings have not yet been classified on knot tables. If the knot is made up of more than one component, I use LinkInfo [32] and the HOMFLY polynomial of the link to manually determine which link I have. In this case I am limited by the fact that links of two components have only been classified up to 11 crossings and for more components even less crossings. I then manually compute the linking number of these knots if they are made up of one component.

After this, I again look to the reduced projection of the knot I have and determine it's tangle notation as inputted on KnotPlot. I start by using the Conway notation

[33] if known of the knot. If the knot exactly matches the Conway notation I note this down. If I need to mirror the notation to produce the knot I note this down, also noting mirrored on my database to keep track of handedness.

From this data we will be able to read off which knots are actually the same. We will get an idea of how many crossings a typical knot has and see how many crossings do not affect the knot mathematically, ie, which are just Reidemeister moves. We will also be able to see which knot has the most uses or appears most often.

3.2 Experiments

Using experiments we aim to test the results learned from the Knot Database.

I plan to run transmission chain experiments to evaluate the effect of different teaching scenarios on the learning of knot tying. The participants in the experiment will be divided into three groups, each given different instructions in which to pass along the knot tying information. Each group will be taught how to tie a knot by an expert. The knot the participants are shown will have a practical purpose and the emphasis will be placed on tying a knot for this purpose, not necessarily copying the knot they are shown. One group will be told they are only allowed to show the next person in the chain the finished knot, and so the next person in the chain will only have this information to help them tie their knot. One group will be told they are only allowed to demonstrate how to tie the knot to the next person in the chain, they must do this silently and only demonstrate once. The third group will be told they are allowed to demonstrate and explain how to tie the knot to the next person in their chain. They will be only allowed to demonstrate once but allowed to answer any questions the learner may have. In this way we will have three separate treatments completing the same task. The first, product only treatment reflects the task of trying to tie a knot observed in its finished product, such as seeing an already tied shoelace, and the practicalities of trying to replicate that with no information. The second, demonstration only treatment reflects the task of trying to tie a knot after observing someone else tie it but with no formal training, such as seeing another tie their shoelaces. The final treatment demonstrates full teaching in knot tying. As each person is not going to be a full expert when they are passing information on to the next participant the experiments won't fully model the real world but can be used to give us a good idea about knot learning. As the emphasis in these experiments will be based on tying a knot for a given purpose we will also be able to see if completing the task outweighs copying what is shown or not.

I also plan to run a transmission chain experiment into the effect of different knot projections on knot replication. From a mathematical point of view, all knots will be the same, that is they will be able to be deformed into each other through a sequence of Reidemeister moves, but they will look different. Participants will be split into three different groups and each group given a different projection of this knot. The participants will transmit the information throughout their group using the most faithful method found in the previous experiments. The emphasis for all participants will be on replicating the knot they are shown. As the knots in all three treatments will be technically the same the point of this experiment will be to see if all chains end with the same knot, whether they all look the same or not. This will give us an idea of whether the appearance of a knot affects its replicability.

I would also like to run a virtual task in which participants are shown knots and asked to replicate them virtually. The task will be available to users directly through their computer or smartphone. As the task is virtual it will be available to a wide range of users and allow us to collect a lot of data quickly. As the task is virtual it won't give us a complete answer to which knots are easiest to replicate but will give us an idea.

The ethics of these experiments will be considered for each individual experiment.

Chapter 4

Timescale

My research is to be completed between October 2014 and October 2018 as follows;

October 2014 - June 2016 - collection of data for Knot Database.

June 2014 - June 2016 - planning and running experiments.

June 2016 - June 2017 - analysis of data from experiments and database with opportunity for more experiments if needed.

June 2017 - December 2017 - first draft.

January 2018 - October 2018 - final write up.

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