Math 167: Mathematical Game Theory – Homework 8

Due: March 3, 2017

Exercise 1.

(1) Show that the map in Exercise 5.2 (from the book of Karlin and Peres, page 112) has no fixed point.

(2) Understand first Lemma 5.2.11(ii) (from the book of Karlin and Peres, page 105), then solve Exercise 5.3. (from the book of Karlin and Peres, page 113).

Exercise 2.

Recall the notion of *correlated equilibrium.* Find all correlated equilibria in the game of *prisoner's dilemma.*

Exercise 3.

(1) Show that in any 2 person general sum game, (where PI has $m \in \mathbb{N}$ strategies and PII has $n \in \mathbb{N}$ strategies) if $(x, y) \in \Delta_m \times \Delta_n$ is a Nash equilibrium, then $c \in \mathbb{R}^{m \times n}$ defined as

$$c_{ij} = x_i y_j, \ i \in \{1, \dots, m\}, \ j \in \{1, \dots, n\}$$

is a correlated equilibrium. *Hint:* you may work by contradiction.

(2) Show that for any 2 person general sum game, convex combination of Nash equilibria produces correlated equilibria. That is, if for a finite number of Nash equilibria $(x^1, y^1), \dots, (x^k, y^k)$ and any $(\lambda_1, \dots, \lambda_k)$ such that $\lambda_i \geq 0$ and $\sum_{i=1}^k \lambda_i = 1$, then

$$(x, y) := \sum_{i=1}^k \lambda_i (x^i, y^i),$$

corresponds to a correlated equilibrium as in (1).