# Math 167: Mathematical Game Theory – Homework 3

Due: January 27, 2017

**Exercise 1** (Partisan subtraction game)**.**

We consider the partisan version of the subtraction game, as introduced during the lectures. PI may remove 1 or 4 chips and PII may remove 2 or 3 chips from a pile of positive number of chips. They take turns, and wins who removes the last chip. Determine the value of the function $W$ (which describes who wins at each game state, i.e. $W : \mathbb{N} \cup \{0\} \times \{I, II\} \to \{I, II\}$) for each $(x, i) \in \{0, \ldots, 8\} \times \{I, II\}$.

Describe the value $W(n, i)$ for arbitrary $n \in \mathbb{N}$ and $i \in \{I, II\}$.

**Exercise 2.**

Consider the *misère* version of the impartial subtraction game, i.e. players have to remove up to 4 chips, but who removes the last chip, loses. Is is possible to say that all game positions are either in the sets $N$ or $P$ in this case? For all $n \in \mathbb{N} \cup \{0\}$ describe, whether $n \in P$, $n \in N$ or none of them. Justify your answers.

**Exercise 3.**

Exercise 1.4 from the book of Karlin and Peres (page 32).

*Bonus question*: solve the same question supposing *misère* play as well. (The bonus points can be used for other homework papers having not full scores).

**Exercise 4.**

Exercise 1.6 from the book of Karlin and Peres (page 32).