1 Express each of the following fractions in lowest possible terms, i.e. write in the form $\dfrac{m}{n}$, where m, n are integers with no common factors.
(i) $\dfrac{24}{32}$ (ii) $\dfrac{76}{116}$ (iii) $\dfrac{168}{448}$ (iv) $\dfrac{63-6}{28+48}$ (v) $\dfrac{13}{48} - \dfrac{23}{24}$.
(For example $\frac{72}{132} = \frac{6}{11}$.)

2 Expand each of the following:
(i) $(a-b)(a+b)$ (ii) $(a-b)(a^2+ab+b^2)$ (iii) $(a-b)(a+b)(a^2+b^2)$
(iv) $(a^2+\sqrt{2}ab+b^2)(a^2-\sqrt{2}ab+b^2)$
(v) $(a+b)(a-b)(a^2+b^2)(a^2+\sqrt{2}ab+b^2)(a^2-\sqrt{2}ab+b^2)$.

3 Expand and simplify the following:
(i) $\dfrac{1}{a+b} + \dfrac{1}{a-b}$ (ii) $\dfrac{1}{a+b}\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$ (iii) $\left(\dfrac{1}{a} + \dfrac{1}{b}\right) \Big/ \left(\dfrac{1}{a-b}\right)$.

4 Simplify each of the following:
(i) $\sqrt{169}$ (ii) $\sqrt{27}$ (iii) $\sqrt[3]{27}$ (iv) $\sqrt[3]{81}$ (v) $\sqrt[4]{4}$ (vi) $\sqrt[10]{32}$.
For example: $\sqrt{8} = \sqrt{(2^3)} = \sqrt{(2^2.2)} = \sqrt{(2^2)}\sqrt{2} = 2\sqrt{2}$, and $\sqrt[3]{16} = \sqrt[3]{2^3.2} = \sqrt[3]{2^3}\sqrt[3]{2} = 2\sqrt[3]{2}$.

5 Simplify each of the following and write in the form $a + b\sqrt{2}$, where a, b are fractions:
(i) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ (ii) $\dfrac{3+\sqrt{2}}{5-\sqrt{2}}$ (iii) $\dfrac{4+3\sqrt{2}}{2\sqrt{2}-1}$ (iv) $\dfrac{3+5\sqrt{2}}{6\sqrt{2}-1} + \dfrac{3-5\sqrt{2}}{6\sqrt{2}+1}$.
For example:

$$\frac{2\sqrt{2}+1}{3\sqrt{2}-1} = \frac{(2\sqrt{2}+1)(3\sqrt{2}+1)}{(3\sqrt{2}-1)(3\sqrt{2}+1)} = \frac{6.2+5\sqrt{2}+1}{9.2-1} = \frac{13+5\sqrt{2}}{17} = \frac{13}{17} + \frac{5}{17}\sqrt{2}.$$

6 Simplify

$$\frac{1}{\sqrt{2}+\sqrt{3}}$$

and also

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}}.$$

7 Find the roots of the following quadratics:
(i) x^2+2x-3 (ii) x^2+5x+6 (iii) x^2-5x+6 (iv) $2x^2+5x+2$.
(Try to do these without relying on the formula each time. For example, to find the roots of $x^2-7x+12$ we may note that $12 = 3 \times 4$, and $7 = 3+4$, so that $x^2-7x+12 = (x-3)(x-4)$.)

8 Show that $x = 1$ is a root of each of the following cubics and hence factorise each one completely.
(i) x^3+2x^2-x-2 (ii) $2x^3-3x^2+1$ (iii) x^3-2x^2-5x+6.

9 Express $(x+1)^6 + (x-1)^6$ and $(x+1)^6 - (x-1)^6$ as polynomials in x.

10 Show that for each integer n the alternating sum of binomial coefficients

$$1 - \binom{n}{1} + \ldots + (-1)^k \binom{n}{k} + \ldots + (-1)^{n-1} \binom{n}{n-1} + (-1)^n$$

is zero. What is the value of the sum

$$1 + \binom{n}{1} + \ldots + \binom{n}{k} + \ldots + \binom{n}{n-1} + 1?$$

11 Write each of the following as a rational multiple of π (i.e. as a fraction times π) and compute their sines and cosines.
(i) $\frac{\pi}{2} - \frac{\pi}{3}$ (ii) $\frac{\pi}{2} + \frac{\pi}{3}$ (iii) $\frac{4\pi}{3} - \frac{7\pi}{2}$ (iv) $\frac{3\pi}{4} + \frac{\pi}{2}$ (v) $\frac{3\pi}{4} - \frac{5\pi}{2}$.

12 Draw the graph of the function $\cos x - \sin x$.

13 Use the formulae for $\cos(A+B)$ and $\sin(A+B)$ to show that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Find (deduce) the corresponding formula for $\tan(A-B)$.

14 Use the formula for $\sin(A+B)$ to show that

$$\operatorname{cosec}(A+B) = \frac{\sec A \sec B}{\tan A + \tan B}$$

and find the corresponding formula for $\operatorname{cosec}(A-B)$.

15 Use the addition formulae to show that:

(i) $\sin(x + \frac{\pi}{2}) = \cos x$ (ii) $\sin(x - \frac{\pi}{2}) = -\cos x$ (iii) $\cos(x + \frac{\pi}{2}) = -\sin x$
(iv) $\cos(x - \frac{\pi}{2}) = \sin x$ (v) $\sin(x + \pi) = -\sin x$ (vi) $\cos(x + \pi) = -\cos x$.

16 For any given value of the integer n, what is the value of $\cos \frac{n\pi}{2}$?

17 For each integer n, find the value of (i) $\dfrac{1 + 2\cos\frac{n\pi}{2}}{2 - \sin\frac{n\pi}{2}}$ (ii) $\dfrac{1 + \sin\frac{n\pi}{2}}{1 + \cos^2\frac{n\pi}{2}}$.

18 Simplify $\sec^2 x \sin 2x \cot x$.

19 Show that (i) $\dfrac{d}{dx}(\arctan x) = \dfrac{1}{1+x^2}$ (ii) $\dfrac{d}{dx}(\arcsin x) = \dfrac{1}{\sqrt{1-x^2}}$.

20 Show that $\arccos x + \arcsin x = \dfrac{\pi}{2}$ for all x such that $-1 \le x \le 1$.

21 Find all solutions to each of the following equations:

(i) $\ln(x+1) = \ln x + 1$ (ii) $e^{2x} = e^2 e^x$ (iii) $\cosh(\ln x^2) = \dfrac{5}{3}$ (iv) $\ln(x+2) = 1 - \ln x$.

22 Write y as a function of x when
(i) $\ln(y+1) - \ln(y+3) = 2x \quad (y > 0)$ (ii) $4e^y + e^{2y} = x \quad (x > 0)$.

23 (i) Find a formula for arcsinh x as a log function.
(ii) Find a formula for arccosh x as a log function.

24 Show that
$$\sinh A - \sinh B = 2\cosh\left(\frac{A+B}{2}\right)\sinh\left(\frac{A-B}{2}\right).$$

25 Show that
$$\tanh(A+B) = \frac{\tanh A + \tanh B}{1 + \tanh A \tanh B}$$

26 (i) Show that
$$\frac{d}{dx}\text{arcsinh}\, x = \frac{1}{\sqrt{1+x^2}}.$$
(ii) Find a similar formula for $\frac{d}{dx}\text{arccosh}\, x$.

27 Show that
(i) $\text{arccoth}\, x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ for $|x| > 1$;
(ii) arcsech $x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ for $0 < x < 1$.

28 Find the derivatives of the following functions:

(i) $(x^2 + \ln x)x^3$	(ii) $(\ln x)\cos(x^2+1)$	(iii) $(1 - x + x^2)e^x$
(iv) $e^{\cos x}(1 + \sin x)$	(v) $e^{\cos(\frac{1}{1+x^2})}$	(vi) $\text{arcsech}\, x$
(vii) x^x	(viii) $\ln(\arctan(1+x))$	(ix) $\ln(\arctan(1 + e^{\cos 2x}))$.

29 Find the indefinite integrals of each of the following functions by using substitutions:
(i) $\dfrac{\ln x}{x}$ (ii) $\dfrac{x}{\sqrt{1-x^4}}$ (iii) $x\sqrt{1-x^2}$ (iv) $\tan x \ln(\cos x)$.

30 Find the indefinite integrals of each of the following functions by using integration by parts:
(i) $x^2 \sin x$ (ii) $(\ln x)^3$ (iii) $\dfrac{\ln(\ln x)}{x}$ (iv) (harder) $\sec^3 x$.

31 Compute each of the following definite integrals using the hints suggested:
(i) $\displaystyle\int_{-\pi}^{\pi} x \sin x \, dx$ (by parts) (ii) $\displaystyle\int_{-\pi}^{\pi} x \cos x \, dx$ (iii) $\displaystyle\int_{-\pi}^{\pi} x \sin 2x \, dx$ (by parts)
(iv) $\displaystyle\int_{-\pi}^{\pi} x \sin nx \, dx$ (by parts), n a +ve integer (v) $\displaystyle\int_{-\pi}^{\pi} x^2 \cos x \, dx$ (by parts twice)
(vi) $\displaystyle\int_{-\pi}^{\pi} x^2 \cos nx \, dx$ (by parts twice), n a positive integer (vii) $\displaystyle\int_{-\pi}^{\pi} x^2 \sin x \, dx$
(viii) $\displaystyle\int_{-\pi}^{\pi} \cos 2x \cos 3x \, dx$ (addition formula)
(ix) $\displaystyle\int_{-\pi}^{\pi} \cos^2 x \cos 2x \, dx$ (use $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$).

32 Find the indefinite integrals of each of the following:

(i) $\frac{1}{x^2+x+1}$ (ii) $x(\cos(x^2)+e^x)$ (iii) $\sin 3x \cos 7x$ (iv) $(\sec x \tan x)^2$

(v) $\frac{1}{\sqrt{x+1}+\sqrt{x-1}}$ (vi) $\frac{x}{(5x^2+1)^2}$ (vii) $x \ln x$ (viii) $\frac{1}{x \ln x}$ (ix) $\frac{\tanh^{-1} x}{1-x^2}$

(x) $\frac{1}{x^2+2x+5}$

33 Find the indefinite integrals of

(i) $\frac{1}{1+x^2}$ (ii) $\frac{x}{1+x^2}$ (iii) $\frac{x^2}{1+x^2}$ (iv) $\frac{x^3}{1+x^2}$

34 Use the method of partial fractions to find the indefinite integrals of each of the following functions:

(i) $\frac{x+4}{(x+1)(x-2)}$ (ii) $\frac{1}{(x-1)(x-2)(x-3)}$ (iii) $\frac{6x^2+3x+17}{(x-1)(x^2+2x+10)}$

(iv) $\frac{x+1}{x^2(x-1)}$ (v) $\frac{1}{x(x^2-x+1)}$ (vi) $\frac{x^4+9}{x^2(x^2+9)}$ (vii) $\frac{x+1}{x^4+1}$

(viii) $\frac{x-1}{x^2(x^2+x+1)^2}$.

35 (i) If $I_n = \int \tan^n x \, dx$, show that $I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$.
Hence find $\int \tan^5 x \, dx$ and $\int \tan^6 x \, dx$.
(ii) If $I_n = \int (\ln x)^n \, dx$, show that $I_n = x(\ln x)^n - nI_{n-1}$.
Hence find $\int (\ln x)^6 \, dx$ and $\int (\ln x)^7 \, dx$.
(iii) If $I_{m,n} = \int \cos^m x \sin nx \, dx$, show that $(m+n)I_{m,n} = -\cos^m x \cos nx + mI_{m-1,n-1}$.
Hence find $\int \cos^5 x \sin 3x \, dx$.
Hint: the equation $\cos nx \sin x = \sin nx \cos x - \sin(n-1)x$ is useful for (iii).

36 Use reduction formulae to find the indefinite integrals of each of the following functions:

(i) $\sin^9 x$ (ii) $\text{cosec}^4 x$ (iii) $\frac{1}{(1+x^2)^7}$.

37 Express each of the following complex numbers in the form $a + \mathrm{i}b$ with a, b real:

(i) $(3+4\mathrm{i})(2+\mathrm{i})$, (ii) $\frac{(3+4\mathrm{i})}{(2+5\mathrm{i})}$, (iii) $\frac{(3+\mathrm{i})}{(2-3\mathrm{i})}$, (iv) $\frac{2-3\mathrm{i}}{2+3\mathrm{i}}$,

(v) $\frac{(1+2\mathrm{i})(5+\mathrm{i})}{(2-3\mathrm{i})(1-\mathrm{i})}$, (vi) $\frac{1+5\mathrm{i}}{1-\mathrm{i}} + \frac{1-5\mathrm{i}}{1+\mathrm{i}}$, (vii) $\frac{2\cos(\pi/3)+2\mathrm{i}\sin(\pi/3)}{\cos(\pi/6)+\mathrm{i}\sin(\pi/6)}$.

38 If $w = \frac{4+3\mathrm{i}}{5+12\mathrm{i}}$, show that $|w| = \frac{5}{13}$ and $\mathrm{Re}\, w = \frac{56}{169}$.

39 Without using a calculator, find the modulus and argument of each of the following:

(i) $2 - i2\sqrt{3}$, (ii) $-1+\mathrm{i}$, (iii) $-\sqrt{3}-\mathrm{i}$, (iv) $\frac{1}{2+\mathrm{i}} + \frac{1}{2-\mathrm{i}}$,

(v) $\frac{1}{2+\mathrm{i}} - \frac{1}{2-\mathrm{i}}$, (vi) $\frac{2-i2\sqrt{3}}{1+i}$ (Hint: Use (i)).

40 If $w = \dfrac{z-1}{z+1}$, show that $\mathrm{Re}(w) = \dfrac{|z|^2-1}{|z|^2+2\mathrm{Re}\, z+1}$ and $\mathrm{Im}(w) = \dfrac{2\mathrm{Im}\, z}{|z|^2+2\mathrm{Re}\, z+1}$.

41 Show that the points $z \neq -1$ such that $\mathrm{Re}(1/(1+z)) \geq 1/2$ make up a circular disc (with one point missing – which one?) on the Argand diagram. Find its centre and radius.

42 Without using a calculator, find z^{25} when z is equal to:
(i) $\dfrac{1}{\sqrt{2}}(1+\mathrm{i})$, (ii) $\sqrt{3}+\mathrm{i}$, (iii)$\sqrt{3}-\mathrm{i}$.
(For the last two, finding the real and imaginary parts as a suitable power of 2 is fine.)

43 The points $2+3\mathrm{i}$ and $1-4\mathrm{i}$ are diagonally opposite vertices of a square in the Argand diagram. Find the other vertices.

44 The points $2+3\mathrm{i}$ and $1-4\mathrm{i}$ are diagonally opposite vertices of a regular hexagon in the Argand diagram. Find the other vertices.

45 A triangle in the Argand diagram has its vertices at the points $a = 1+7\mathrm{i}$, $b = 2-\mathrm{i}$, $c = 3+5\mathrm{i}$. Rotation through a certain angle θ about the origin moves the vertices to the points a', b', c' respectively and one of these points is $5+5\mathrm{i}$. Which one, and what are the other points?

46 Show that
(i) $\sin^5\theta = \dfrac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$.
(ii) $\sin^6\theta = -\dfrac{1}{32}(\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10)$.

47 Express $\sin^6\theta\cos 3\theta$ in the form $a_0 + a_1\cos\theta + \cdots + a_r\cos r\theta$ where the a_i are real constants.

48 (i) Express $\cos 4\theta$ as a polynomial in $\sin\theta$.
(ii) Using (i) find a polynomial equation with integer coefficients which is satisfied by $\sin\left(\dfrac{\pi}{12}\right)$.
Hint: $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.

49 Show that, for all complex numbers z and w,

$$\cos(z+w) = \cos z\cos w - \sin z\sin w$$

and deduce that, for all complex numbers z,

$$\cos^2 z + \sin^2 z = 1 \quad \text{and} \quad \cos^2 z - \sin^2 z = \cos 2z.$$

50 Show that $\cos z = \cos x\cosh y - \mathrm{i}\sin x\sinh y$ and that $|\cos z|^2 = \cos^2 x + \sinh^2 y$.

51 Calculate $\mathrm{Re}(\tan z)$ as a real function of x and y.

52 Find the modulus and argument of each of the following:
(i) $e^{\frac{2-3\mathrm{i}}{2+3\mathrm{i}}}$ (ii) $\sin((1+\mathrm{i})\frac{\pi}{2})$.

53 Find all the solutions to each of the following equations:
(i) $e^z = e^{-1+i\pi}$, (ii) $\sin z = 0$, (iii) $\cosh z = 0$.

54 Find all the solutions to the equation $e^{\frac{1}{z}} = \frac{e(1+\mathrm{i})}{\sqrt{2}}$. Which one has the greatest modulus?

55 Find all the solutions to each of the following equations:
(i) $e^z = e^{2+\mathrm{i}}$ (ii) $\cos z + \sin z = 2$ (iii) $\cosh z + 2\sinh z = 1$.

56 Find all values of z such that $\cos z = 2$.

57 Find all values of z such that $\tan z = 1$.

58 Find all the solutions to each of the following equations:
(i) $z^4 = -1 - \mathrm{i}\sqrt{3}$, (ii) $z^3 = 1 - \mathrm{i}$, (iii) $z^8 + 4z^4 + 16 = 0$, (iv) $(z^2 - 1)^3 = 1$,
(v) $z^2 + 4z + 4 + 2\mathrm{i} = 0$.

59 Find all six roots of the equation $z^6 + 2z^3 + 2 = 0$. *Hint:* First find z^3.

60 Find all the roots of $z^8 + 1 = 0$ and plot them on the Argand diagram.

61 Express $\frac{z^7 - 1}{z - 1}$ as a product of quadratic polynomials with real coefficients.

62 Show that $\cos\frac{2\pi}{5} = \frac{-1+\sqrt{5}}{4}$. (Don't use calculators. Think about $w = e^{\frac{2\pi\mathrm{i}}{5}}$).

63 If $\zeta = \exp\frac{2\pi\mathrm{i}}{7}$ show that $1 + \zeta + \zeta^2 + \zeta^3 + \zeta^4 + \zeta^5 + \zeta^6 = 0$.
Using the fact that $\zeta^r + \zeta^{-r} = 2\cos(2\pi r/7)$, show that

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7} = -\frac{1}{2}.$$

Also show (this is harder!) that

$$\left(\sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}\right)^2 = \frac{7}{4}.$$

64 Show that $\sqrt{2} + \sqrt{3}$ and $\sqrt{2} + \sqrt{3} + \sqrt{5}$ are both irrational.

65 Show that $\sqrt[3]{2}$ is irrational.

66 If a, b, c, d are distinct rational numbers and $a + \sqrt{b} = c + \sqrt{d}$ show that $\sqrt{b}$ and $\sqrt{d}$ are both rational.

67 Suppose a and b are rational numbers with b positive. Show that (i) if $(a + \sqrt{b})^3$ is rational then so is $\sqrt{b}$, and (ii) the same is true if $(a + \sqrt{b})^4$ is rational and $a \neq 0$.

68 Find the limits (where they exist) of each of the following as x tends to 0:
(i) $\dfrac{\tan x}{x}$ (ii) $\dfrac{x^3+x}{\sin x}$ (iii) $x\cos x$ (iv) $x\sin\dfrac{1}{x}$ (v) $\dfrac{\sin x}{1-\cos x}$ (vi) $\dfrac{x^2}{1-\cos x}$.

69 Find the limit as x tends to 1 of $(x^2-x-2)/(x^2+x-6)$.

70 Find the limit as x tends to 2 of $(x^2-x-2)/(x^2+x-6)$.

71 Find the limits of each of the following as x tends to $+\infty$:
(i) $\dfrac{x^2+1}{x^2+2}$ (ii) $\dfrac{\sin x}{x}$ (iii) $2^{-x}\sin(x^2)$ (iv) $\tanh x$ (v) $\dfrac{\tanh x}{x}$ (vi) $\dfrac{(1+x^3)}{\cosh x}$.

72 If $f(x)=\dfrac{x^3+x^2-2}{x^3-x^2+3x-3}$, find $\lim_{x\to 0} f(x)$, $\lim_{x\to 1} f(x)$ and $\lim_{x\to -1} f(x)$.

73 Find the limits of each of the following as x tends to 0:
(i) $\dfrac{e^x-1}{x}$ (ii) $\dfrac{\tan x^2}{x}$ (iii) $\dfrac{\tan x}{\sqrt{x}}$ (iv) $\dfrac{\sin 7x}{x}$ (v) $\dfrac{\sin 7x}{\sin x}$.

74 Find $\lim_{x\to\pi/2}(x-\pi/2)\tan x$.

75 Show that each of the following functions is continuous at $x=0$:
(i) $|x|\sin x$ (ii) $\sin(|x|)$ (iii) $\sin(x+|x|)$ (iv) $\dfrac{x}{1+|x|}$.

76 How must $f(1)$ be defined in order that $f(x)$ is continuous at $x=1$ in each of the following cases?
(i) $f(x)=\dfrac{x^3+5x^2+2x-8}{x-1}$ (ii) $f(x)=\dfrac{\sin\pi x}{x-1}$ (iii) $f(x)=\dfrac{\sin^2\pi x}{x-1}$.

77 Decide where each of the following functions is discontinuous:
(i) $\dfrac{1}{x^2-1}$ (ii) $\dfrac{x^2+x-2}{x^2-1}$ (iii) $\dfrac{\sin x}{x(x+1)}$ (iv) $\dfrac{x}{\sin x}$.

78 If $f(x)=\cos x$, prove that $f'(x)=-\sin x$.

79 Show that, for each integer $n\geq 1$, the function $f(x)=x|x|^n$ is differentiable for all x and find its derivative.

80 Show that if $f(x)$ is continuous at $x=0$ then $f(x)\sin x$ is differentiable at $x=0$. Hence show that $|x|\sin x$ is differentiable everywhere and find its derivative.

81 Find the regions of the real line where the function $f(x)=\dfrac{x+1}{x^2+1}$ is
(a) increasing, (b) decreasing, and draw the graph of $f(x)$.

82 Find the regions of increase and the regions of decrease of $x^3-9x^2+24x+2012$.

83 Determine the maximum value of each of the following functions on the interval $1\leq x\leq 2$:
(i) x^2-3x+1 (ii) $x\sin x$ (iii) $\dfrac{e^x}{1+x^2}$.

84 Determine the maximum and minimum values of $x^4 - 4x^3 + 1$ on the interval $-1 \le x \le 1$ and the points at which they are attained.

85 Determine the maximum and minimum values of $x^6 - 6x + 1$ on the interval $-2 \le x \le 2$ and the points at which they are attained.

86 Find the maximum values (if any) and the points at which they are attained of each of the following:
(i) $x^4 - 2x^2$ on $\frac{1}{3} \le x \le \frac{4}{3}$ (ii) $x^4 - 2x^2$ on $-\frac{1}{3} \le x \le \frac{4}{3}$ (iii) $x^4 - 2x^2$ on $-\frac{1}{3} \le x \le 2$
(iv) $x^4 - 2x^2$ on $0 \le x \le 1$ (v) $1 - |1 - x^2|$ on $0 \le x \le \sqrt{2}$ (vi) $\frac{x}{x^2+1}$ on $x \ge 0$
(vii) $\frac{x}{x+1} \cos\left(\frac{1}{x}\right)$ on $x \ge 1$.

87 Find the least value (if any) attained by each of the following functions of x when x lies in the given range:
(i) $\cos x - 3x$ for $-2 \le x \le 4$,
(ii) $2x^4 - 8x^3 + 9x^2 + 3$ for $-\infty < x < \infty$,
(iii) $x(x-1)(x-3)$ for $-2 \le x \le 4$.

88 Show that $e^x > 1 + x$ for all real $x \ne 0$.

89 Show that if a, b, c are real constants then $f(x) = x^3 + ax^2 + bx + c$ is strictly increasing everywhere if and only if $a^2 \le 3b$.

90 Suppose that $f(x)$ is differentiable for $0 < x < 10$ and continuous for $0 \le x \le 10$. If $f(x)$ is zero at 3 points of $0 \le x \le 10$, show that $f'(x)$ is zero at at least 2 points of $0 < x < 10$.

91 Use Rolle's theorem to show that the equation $5x^4 - 6x + 1 = 0$ does not have more than 2 real solutions.

92 Use L'Hopital's Rule to calculate the following limits:
(i) $\lim_{x\to 0} \frac{\tan x}{x^2 + \sin x}$ (ii) $\lim_{x\to 1} \frac{x^7 - 10x^6 + 4x^5 + 2x^3 + 4x - 1}{x^{23} + 2x^{17} + x^{11} - 7x^9 + 4x^2 - 1}$
(iii) $\lim_{x\to e} \frac{\ln(\ln x)}{\ln x - 1}$ (iv) $\lim_{x\to 0} \frac{\sin e^{x^2} - \sin 1}{x}$.

93 Use L'Hopital's rule (possibly repeatedly) to find the limit as x tends to $\pi/2$ of $(1 - \sin x)/\cos^2 x$.

94 Use l'Hopital's Rule (possibly repeatedly) to calculate the following limits:
(i) $\lim_{x\to 0} \frac{\cot 5x}{\cot 3x}$ (ii) $\lim_{x\to 0} \frac{1 - \cos x}{x + x^3}$ (iii) $\lim_{x\to 0} \frac{1}{\sin x} - \frac{1}{x}$ (iv) $\lim_{x\to\infty} \frac{x^2}{e^x}$.

95 Find the following limits: (i) $\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}}$ (ii) $\lim_{x\to\infty} (e^{2x} - x)^{\frac{1}{x}}$.

96 Use induction to prove that $\sum_{k=1}^{n} k^3 = \frac{1}{4}\left(n(n+1)\right)^2$.

97 Use $\delta - \epsilon$ method to show that $f(x) = x + 3$ has, as $x \to 1$, the limit $\lim_{x\to 1} f(x) = 4$

98 Use $\delta - \epsilon$ method to show that $f(x) = x^2 + 1$ has, as $x \to 1$, the limit $\lim_{x\to 1} f(x) = 2$

99 Use $\delta - \epsilon$ method to show that $f(x) = \frac{x^2+x+1}{2x}$ is continuous at $x = 1$.