Q35 - continued

In the similar way

$$\hat{x}^3 = \hat{x}^2\hat{x} = \frac{i}{2m\omega^3}\frac{1}{\sqrt{2m}}\left(-\hat{a}^{+2} - \hat{a}^2 + 2\hat{H}\right)\left(\hat{a}^+ - \hat{a}\right) =$$

$$= \frac{i}{(2m)^{3/2}\omega^3}\left[-\hat{a}^{+3} - \hat{a}^2\hat{a}^+ + 2\hat{H}\hat{a}^+ + \hat{a}^{+2}\hat{a} + \hat{a}^3 - 2\hat{H}\hat{a}\right]$$

so $\langle p|\left(-\hat{a}^{+3} - \hat{a}^2\hat{a}^+ + 2\hat{H}\hat{a}^+ + \hat{a}^{+2}\hat{a} + \hat{a}^3 - 2\hat{H}\hat{a}\right)|s\rangle =$

$$= \left(\frac{\hbar}{2m\omega}\right)^{3/2}\left[-\langle p|s-3\rangle\sqrt{s(s-1)(s-2)} - \langle p|s+1\rangle\sqrt{s^2(s+1)} + 2\langle p|s-1\rangle\sqrt{s}\left(s-\tfrac{1}{2}\right)\right.$$

$$\left. + \langle p|s-1\rangle\sqrt{(s+1)^2 s} + \langle p|s+3\rangle\sqrt{(s+1)(s+2)(s+3)} - \langle p|s+1\rangle\sqrt{s+1}\left(s+\tfrac{3}{2}\right)\right]$$

and then rewrite all this using

$$\langle p|r\rangle = \delta_{pr} \quad \text{etc.}$$