Quantum Mechanics – Solutions

Q31 Let $\hat{P} = [\hat{J}_z, \hat{J}_x\hat{J}_y + \hat{J}_y\hat{J}_x] = \hat{J}_x[\hat{J}_z, \hat{J}_y] + [\hat{J}_z, \hat{J}_x]\hat{J}_y +$

$$+ \hat{J}_y[\hat{J}_z, \hat{J}_x] + [\hat{J}_z, \hat{J}_y]\hat{J}_x$$

But $\quad [\hat{J}_x, \hat{J}_y] = i\hbar\hat{J}_z$

so $\quad \hat{P} = 2i\hbar\left(\hat{J}_y^2 - \hat{J}_x^2\right).$

Take $\quad \hat{J}_z|\psi\rangle = b|\psi\rangle \qquad \hat{J}^2|\psi\rangle = a|\psi\rangle$

Then $\quad \langle\hat{J}_x^2 + \hat{J}_y^2\rangle = \langle\psi|\hat{J}^2 - \hat{J}_z^2|\psi\rangle = a - b^2$

$$\langle\psi|\hat{P}|\psi\rangle = \langle\psi|[J_z, \hat{J}_x\hat{J}_y + \hat{J}_y\hat{J}_x]|\psi\rangle = \langle\psi|\left(\hat{J}_x\hat{J}_y + \hat{J}_y\hat{J}_x\right)|\psi\rangle(b-b)$$

$$= 0$$

But $\langle\psi|\hat{P}|\psi\rangle = \left(\langle\hat{J}_y^2\rangle - \langle\hat{J}_x^2\rangle\right)2i\hbar \quad \therefore \langle\hat{J}_y^2\rangle = \langle\hat{J}_x^2\rangle$

$$\therefore \quad \langle\hat{J}_x^2\rangle = \langle\hat{J}_y^2\rangle = \frac{a-b^2}{2}.$$

~~But~~ note that $\quad i\hbar\hat{J}_x = [\hat{J}_y, \hat{J}_z]$

so $\quad \langle\psi|\hat{J}_x|\psi\rangle = \frac{1}{i\hbar}\langle\psi|[\hat{J}_y, \hat{J}_z]|\psi\rangle = 0 \qquad$ as $\quad \hat{J}_z|\psi\rangle = b|\psi\rangle$

So $\qquad \Delta J_x = \sqrt{\langle\hat{J}_x^2\rangle - \langle\hat{J}_x\rangle^2} = \sqrt{\langle\hat{J}_x^2\rangle} = \sqrt{\frac{a-b^2}{2}}$

$$\Delta J_y = \Delta J_x.$$