Q.23 State $\begin{pmatrix} a \\ b \end{pmatrix}$ $\quad a, b \in \mathbb{C}$

then if $\begin{pmatrix} a \\ b \end{pmatrix} = |A\rangle$

Define scalar product $\langle B|A\rangle =$

$= (c^* a + d^* b)$ where $\langle B| = (c^*, d^*)$

i.e. $|A\rangle \longrightarrow \langle A|$ corresponds to "hermitian conjugation" of $\begin{pmatrix} a \\ b \end{pmatrix}$

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linear operators are ~~linear~~ matrices

$$\hat{A}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \alpha, & \beta \\ \gamma, & \delta \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

observables are linear operators that are selfadjoint $\therefore$ $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}^{\dagger} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$

But $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}^{\dagger} = \begin{pmatrix} \alpha^* & \gamma^* \\ \beta^* & \delta^* \end{pmatrix}$ $\therefore$ α real, δ real $\quad \beta = \gamma^*$

Space of such (hermitian) matrices is spanned by 4 matrices $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Any of the last 3 commutes with I but not with each other. So can diagonalize (and have common eigenstates) of I and say $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\quad I \sim \hat{S}^2$ $\quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \sim \hat{S}_z$ $\quad$ remaining two stand for $\hat{S}_x$ & $\hat{S}_y$