Q10 $\hat{A} = \frac{d^2}{dx^2} - x^2$ $\qquad u = \exp(-x^2/2)$

$$\hat{A}u = \frac{d^2}{dx^2}\exp\left(-\frac{x^2}{2}\right) - x^2\exp\left(-\frac{x^2}{2}\right) =$$

$$= \frac{d}{dx}\left(-x\exp\left(-\frac{x^2}{2}\right)\right) - x^2\exp\left(-\frac{x^2}{2}\right) = -\exp\left(-\frac{x^2}{2}\right)$$

so $\hat{A}u = -u$ $\quad$ ∴ eigenvalue $= -1$.

Q11 a) $[\hat{A},\hat{B}] = \hat{C}$, $\qquad [\hat{A},\hat{C}] = [\hat{B},\hat{C}] = 0$

∴ $[\hat{A},\hat{B}^n] = ?$ Use induction $[\hat{A},\hat{B}] = [\hat{A},\hat{B}]$

Assume $[\hat{A},\hat{B}^{n-1}] = (n-1)\hat{B}^{n-2}[\hat{A},\hat{B}]$ ie.

$$\hat{A}\hat{B}^{n-1} - \hat{B}^{n-1}\hat{A} = (n-1)\hat{B}^{n-2}[\hat{A},\hat{B}]$$

$$\hat{A}\hat{B}^n - \hat{B}^n\hat{A} = (\hat{A}\hat{B}^{n-1})\hat{B} - \hat{B}^n\hat{A} = \left(\hat{B}^{n-1}\hat{A} + (n-1)\hat{B}^{(n-2)}[\hat{A},\hat{B}]\right)\hat{B}$$

$$-\hat{B}^n\hat{A} = \hat{B}^{n-1}\hat{A}\hat{B} + (n-1)\hat{B}^{n-1}[\hat{A},\hat{B}] - \hat{B}^n\hat{A} = \hat{B}^{n-1}[\hat{A},\hat{B}]$$

$$+(n-1)\hat{B}^{n-1}[\hat{A},\hat{B}] = n\hat{B}^{n-1}[\hat{A},\hat{B}]$$ as required.

b) $[\hat{A}, e^{\hat{B}}] = e^{\hat{B}}[\hat{A},\hat{B}]$?

$$[\hat{A}, e^{\hat{B}}] = \left[\hat{A}, \sum_{n=0}^{\infty}\hat{B}^n\frac{1}{n!}\right] = \sum_{n=0}^{\infty}\frac{1}{n!}[\hat{A},\hat{B}^n] = \sum_{n=0}^{\infty}\frac{1}{n!}n\hat{B}^{n-1}[\hat{A},\hat{B}]$$

$$= \sum_{k=0}^{\infty}\frac{B^k}{k!}[\hat{A},\hat{B}] = e^{\hat{B}}[\hat{A},\hat{B}].$$