Quantum Mechanics ~ Solutions

3a) continued.

$$\lambda=-1; \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -v_1 \\ -v_2 \\ -v_3 \end{pmatrix} \qquad \begin{array}{l} v_2 = -v_1 \\ v_3 = -v_3 \quad \therefore \ v_3 = 0 \end{array}$$

eigenvector $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ normalised $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}\frac{1}{\sqrt{2}}$.

$$\lambda=1 \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \quad \therefore \quad \begin{array}{l} v_1 = v_2 \\ v_3 = v_3 \end{array}$$

can take

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\frac{1}{\sqrt{2}}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0 \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 0 \qquad \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 0$$

b) $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}\underline{v} = \lambda \underline{v}$

$$\det\begin{pmatrix} 2-\lambda & 4 \\ 1 & 2-\lambda \end{pmatrix} = 0 \qquad (2-\lambda)^2 - 4 = 0$$

$$\therefore \ \lambda = 0, \ \lambda = 4$$

$$\lambda=0 \quad \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \qquad 2v_1 + 4v_2 = 0 \qquad v_2 = -\tfrac{1}{2}v_1$$

eigenvector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}\frac{1}{\sqrt{3}}$

clearly

$$\lambda=4 \quad \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 4v_1 \\ 4v_2 \end{pmatrix} \quad \therefore \quad v_2 = \tfrac{1}{2}v_1 \quad \therefore \quad \begin{pmatrix} 2 \\ 1 \end{pmatrix}\frac{1}{\sqrt{3}} \qquad (-2,1)(2,1) = 1-4 = -$$

no orthogonality.