Quantum Mechanics – Solutions

1.

$$(A+B)(A-B) = AA + BA - A\cdot B - B\cdot B$$

if $A^2 - B^2 = (A+B)(A-B) = A^2 - B^2 + BA - AB$

$\therefore\ BA - AB = 0 \quad \therefore\ [A, B] = 0$

2.

$$U = e^{iaH}$$

$H = H^{+}\ \therefore$

$$U^{+} = \left(e^{iaH}\right)^{+} = e^{-iaH^{+}} = e^{-iaH}$$

But $U^{-1} = e^{-iaH}$ & $UU^{-1} = \mathbb{1}$ $\therefore$ U is unitary.

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If $U^{-1} = U^{+}$ then

$$UU^{+} = U^{+}U = \mathbb{1} = e^{iaH}\, e^{-iaH^{+}}$$

so $H - H^{+} = \frac{2\pi n}{a}$,

$U = e^{iaH}$

$U^{-1} = e^{-iaH}$

$U^{+} = e^{-iaH^{+}}$

$\therefore\ e^{-iaH} = e^{-iaH^{+}}$

$\therefore\ H^{+} = H + \frac{2n\pi}{a}$

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3. a) $A^{T} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \therefore\ A^{T} = A$

$Av = \lambda v$ $\quad \therefore \det\left[\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right] = \det \begin{pmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{pmatrix} =$

$= (1-\lambda)(\lambda^2 - 1) = 0 \quad \therefore\ \lambda = 1$ two values

$\lambda = -1$