QM Homework Problem 3

(a) Find N so that the state $|\psi\rangle$ with Schrödinger position representation wavefunction

$$\psi(x) = Ne^{-\frac{(x-y)^2}{2a^2}}$$

is normalised, where a and y are real constants.

Sketch the function $|\psi(x)|^2$. [You should be able to read off the value of $\langle x\rangle$ from this graph without any further calculation.]

Using the standard results for Gaussian integration, calculate $\langle x\rangle$, $\langle p\rangle$, Δx and Δp for the state $|\psi\rangle$.

(b) Find the momentum representation wavefunction $\tilde{\psi}(p)$. Now check that $\langle x\rangle$, calculated using $\tilde{\psi}(p)$, agrees with the result in part **(a)**.

Sketch the function $|\tilde{\psi}(p)|^2$. [You should be able to read off the value of $\langle p\rangle$ from this graph without any further calculation.]

[As an additional exercise you can also check that you get the same results for $\langle p\rangle$, Δx and Δp as in part (a).]

(c) Show that the position-momentum uncertainty principle is satisfied for any value of a. Briefly comment on the interpretation of the state $|\psi\rangle$ in the limits $a \to 0$ and $a \to \infty$.