

Q7 continued

$$\text{so } |\psi_t\rangle = e^{-\frac{2it}{\hbar}} \left(\frac{1}{2}(|1\rangle + i|2\rangle) + \frac{1}{2}|1\rangle - \frac{i}{2}|2\rangle \right)$$

$$= \frac{1}{2} \left[e^{-\frac{2it}{\hbar}} + 1 \right] |1\rangle + \frac{i}{2} \left[e^{-\frac{2it}{\hbar}} - 1 \right] |2\rangle$$

Subsequent measurements \hat{A} would give either 1 or 2 (but never 3) with probabilities (relative to each other)

$$\left| \frac{1}{2} (e^{-\frac{2it}{\hbar}} + 1) \right|^2 \quad \text{and} \quad \left| \frac{i}{2} (e^{-\frac{2it}{\hbar}} - 1) \right|^2$$

respectively. i.e. $\left(1 + \cos \frac{2t}{\hbar}\right)^2$ and $\left(1 - \cos \frac{2t}{\hbar}\right)^2$

Thus the probabilities of 1 and 2 are respectively

$$\frac{1}{2} + \frac{\cos \phi}{1 + \cos^2 \phi} \quad \text{and} \quad \frac{1}{2} - \frac{\cos \phi}{1 + \cos^2 \phi}$$

$$\text{where } \phi = \frac{2t}{\hbar}$$

The expectation value of \hat{A} i.e. $\langle \hat{A} \rangle$ is $\sum \text{value} \cdot p$

$$\langle \hat{A} \rangle = \frac{1}{2} + \frac{\cos \phi}{1 + \cos^2 \phi} + 2 \left(\frac{1}{2} - \frac{\cos \phi}{1 + \cos^2 \phi} \right) = \frac{3}{2} - \frac{\cos \phi}{1 + \cos^2 \phi} \quad \text{where } \phi = \frac{2t}{\hbar}$$

The remeasurement would give 2 at those

$$t \quad \text{where} \quad \left| e^{-\frac{2it}{\hbar}} + 1 \right|^2 = 0 \quad \text{i.e.} \quad \cos \phi = -1 \quad \therefore t = \frac{(2n+1)\pi}{2}$$

n integer.