

(67) First diagonalise the Hamiltonian
 clearly $|3\rangle$ is an eigenstate with eigenvalue

$$\text{let } |a\rangle = \alpha|1\rangle + i\beta|2\rangle$$

$$\hat{H}|a\rangle = \alpha\hat{H}|1\rangle + i\beta\hat{H}|2\rangle = \alpha|1\rangle + i\alpha|2\rangle + i\beta|2\rangle + \beta|1\rangle \\ = (\alpha+\beta)|1\rangle + i(\alpha+\beta)|2\rangle$$

For eigenstates $(\alpha+\beta)|1\rangle + i(\alpha+\beta)|2\rangle = \gamma(\alpha|1\rangle + i\beta|2\rangle)$

so $|a_+\rangle = \frac{1}{2}(|1\rangle + i|2\rangle)$ is an eigenstate

$$\hat{H}|a_+\rangle = 2|a_+\rangle$$

and $|a_-\rangle = \frac{1}{2}(|1\rangle - i|2\rangle)$ is another eigenstate

$$\hat{H}|a_-\rangle = 0$$

At $t=0$ \hat{A} is measured and forced to give 1. ^{soon} afterwards the state is $|\psi(t=0)\rangle = |1\rangle$

This state evolves as

$$|\psi_t\rangle = e^{-\frac{i\hat{H}t}{\hbar}} |\psi_0\rangle$$

But $|\psi_0\rangle = |1\rangle = \frac{1}{2} [|a_+\rangle + |a_-\rangle]$

so $|\psi_t\rangle = e^{-\frac{i\hat{H}t}{\hbar}} \left(\frac{1}{2} |a_+\rangle + \frac{1}{2} |a_-\rangle \right) = e^{-\frac{2it}{\hbar}} \frac{1}{2} |a_+\rangle + \frac{1}{2} |a_-\rangle$