Quantum Mechanics – Solutions

Q6

$$V(r) = \frac{A}{r^2} - \frac{B}{r}$$

So the radial equation is

$$-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \left\{\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2} + \frac{A}{r^2} - \frac{B}{r}\right\}R = ER$$

Note that $\quad l(l+1)\frac{\hbar^2}{2m} + A = \mu(\mu+1)\frac{\hbar^2}{2m}$

$$\text{so} \quad \mu(\mu+1) = l(l+1) + A\frac{2m}{\hbar^2}.$$

So the problem looks like the hydrogen atom of angular momentum $l \to \mu$

For the hydrogen atom we have

$$-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{Ze^2}{r}R + \frac{l(l+1)\hbar^2}{2mr^2}R = ER$$

$$\text{then} \quad \lambda = -\frac{Ze^2}{2E}\sqrt{\frac{-2mE}{\hbar^2}} = l + \nu + 1 \leftarrow \text{integer}$$

this has to be an integer.

~~then~~ and so

$$\left(\frac{Ze^2}{2E}\right)^2\frac{-2mE}{\hbar^2} = (l+\nu+1)^2$$

$$\therefore \quad \frac{(Ze^2)^2}{2E} = -\frac{\hbar^2}{m}(l+\nu+1)^2$$