## QM. Solutions

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$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

$$\therefore \; -\frac{d^2\psi}{dx^2} - 2(\mathrm{sech}\,x)\psi = E\psi$$

Put $\phi = \mathrm{sech}\,x = \frac{1}{\mathrm{ch}\,x}$ $\quad \therefore \phi_x = -\frac{\sinh x}{\mathrm{ch}^2 x}$, $\quad \phi_{xx} = -\frac{1}{\mathrm{ch}\,x} + 2\frac{\sinh^2 x}{\mathrm{ch}^3 x}$

So $\frac{1}{\mathrm{ch}\,x} - 2\frac{\sinh^2 x}{\cosh^3 x} - 2\frac{1}{\mathrm{ch}^3 x} = -\frac{1}{\mathrm{ch}\,x}$ $\quad \therefore$ so $\phi(x)$ is a solution with eigenvalue $-1$.

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$$\phi = \exp(ikx)\left[-ik + \tanh x\right]$$

$$\phi_x = ik\,e^{ikx}\left[-ik + \tanh x\right] + e^{ikx}\frac{1}{\mathrm{ch}^2 x}$$

$$\phi_{xx} = -ik^2 e^{ikx}\left[-ik + \tanh x\right] + 2ik\,e^{ikx}\frac{1}{\mathrm{ch}^2 x} - 2e^{ikx}\frac{\mathrm{th}\,x}{\mathrm{ch}\,x}$$

So $k^2\left[-ik + \tanh x\right] - \frac{2ik}{\mathrm{ch}^2 x} + \frac{2\,\mathrm{th}\,x}{\mathrm{ch}\,x} - 2\frac{1}{\mathrm{ch}^2 x}\left[-ik + \tanh x\right]$

$= k^2\left[-ik + \tanh x\right]$

So $\phi = \exp(ikx)\left[-ik + \tanh x\right]$ is a solution with energy $k^2$.

As $x \to +\infty$ $\quad \phi = e^{ikx}\left[-ik + \tanh x\right] \rightarrow e^{ikx}\left[-ik + 1\right]$

as $x \to -\infty$ $\quad \phi \rightarrow e^{ikx}\left[-ik - 1\right]$

So there is no reflection i.e. complete transmission.