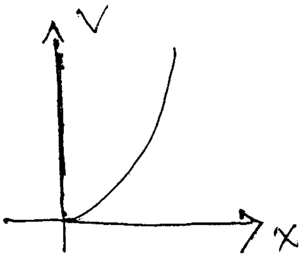
# Quantum Mechanics – Solutions

**Q55.**




$$V(x) = \begin{cases} \infty & x < 0 \\ \frac{1}{2} m\omega^2 x^2 & x > 0 \end{cases}$$

So this a bit like the harmonic oscillator (except that we require $\psi(0) = 0$ and $\psi = 0$ for $x < 0$).

But solutions of the harmonic oscillator are given by $\psi_n = e^{-\frac{1}{2}y^2} H_n(y)$

where $y = \sqrt{\frac{m\omega}{\hbar}}\, x$ and $H_n$ is a Hermite polynomial. $H_n(y)$ is a polynomial of degree $n$ and satisfies $H_n(-y) = (-1)^n H_n(y)$

so $H_{2k+1}(0) = 0$ so our solutions

are $\psi_{2k+1}(x)$ of the harmonic oscillator.
$k = 1, 2, \ldots$ i.e. its odd wave functions

$$E = \frac{\lambda \hbar\omega}{2} = \frac{\hbar\omega}{2}(2n+1) \quad n = 1, 3, \ldots$$

The lowest state has $n = 1$ $\therefore E = \frac{\hbar\omega\, 3}{2}$.

and is given by $\psi_1 = e^{-\frac{1}{2}y^2} H_1(y) = e^{-\frac{1}{2}y^2} y$.

---

Or repeat the discussion given in lecture.

$\therefore \frac{d^2\psi}{dy^2} + (\lambda - y^2)\psi = 0$ $\quad \left[\psi \underset{y\to\infty}{\to} 0 \;\middle|\; \psi(0) = 0\right]$

$\therefore \psi = e^{-\frac{1}{2}y^2} H(y)$ $\underset{H_1, H_3, H_5 \ldots}{\Rightarrow}$ Hermite polynomials.