

Quantum Mechanics - Solutions

42 continued (end)

so after time t the state becomes

$$\begin{aligned}
 |\phi(t)\rangle &= e^{-\frac{iHt}{\hbar}} |\phi\rangle = e^{-\frac{iHt}{\hbar}} \frac{2i}{\sqrt{5}} \left(\frac{-2i|1\rangle + \sqrt{3}|3\rangle}{\sqrt{5}} \right) \\
 &\quad - e^{\frac{iHt}{\hbar}} \frac{i}{\sqrt{5}} \left(\frac{i|1\rangle + 2|3\rangle}{\sqrt{5}} \right) = \\
 &= |1\rangle \left(\frac{4}{5} e^{-\frac{2it}{\hbar}} + \frac{1}{5} e^{\frac{3it}{\hbar}} \right) + |3\rangle \left(\frac{2i}{5} e^{-\frac{2it}{\hbar}} - \frac{2i}{5} e^{\frac{3it}{\hbar}} \right).
 \end{aligned}$$

so from these we can see the measurements of B will give 1 or 3 with probabilities

$$\left| \frac{4}{5} e^{-\frac{2it}{\hbar}} + \frac{1}{5} e^{\frac{3it}{\hbar}} \right|^2 \quad \text{and} \quad \left| \frac{2i}{5} \left(e^{-\frac{2it}{\hbar}} - e^{\frac{3it}{\hbar}} \right) \right|^2$$

$$\text{i.e.} \quad \frac{16 + 1 + 8 \cos \frac{5t}{\hbar}}{25} = \frac{17 + 8 \cos \frac{5t}{\hbar}}{25} \quad \text{and} \quad \frac{8}{25} \left(1 - \cos \frac{5t}{\hbar} \right)$$

respectively.
Hence we will find the values 1 after $\frac{5t}{\hbar} = 2\pi$

$$\langle B \rangle = \frac{(17 + 8 \cos \frac{5t}{\hbar}) + 3 \cdot 8 (1 - \cos \frac{5t}{\hbar})}{25}$$

The max probability of finding 3 is $\frac{16}{25}$
So never certainty!

If original observation gave 2 or 3 in eigenstate of \hat{B} all subsequent measurements will also give 2!