Quantum Mechanics - Solutions

42 continued

to find (E) $\therefore$ $\det\begin{pmatrix}1-E & -2i \\ 2i & -2-E\end{pmatrix} = 0$

$(1-E)(2+E)+4=0$ $\quad E=-3$ or 2

corresponding eigenvectors.

$E=-3$ $\quad -3A = A - 2iB$ $\quad A=\frac{i}{2}B$ $\quad$ to normalise it.

so $|x\rangle = \left(i|1\rangle + 2|3\rangle\right)\frac{1}{\sqrt{5}}$

and for $E=2$ $\quad A=-2iB$

$|x\rangle = \left(-2i|1\rangle + |3\rangle\right)\frac{1}{\sqrt{5}}$

so $|\psi_2\rangle = \left(-2i|1\rangle + |3\rangle\right)\frac{1}{\sqrt{5}}$ is an eigenstate of $\hat{H}$ corresponding to $E=2$

$|\psi_{-3}\rangle = \left(i|1\rangle + 2|3\rangle\right)\frac{1}{\sqrt{5}}$ - - - $E=-3$.

$|2\rangle$ is an eigenstate of $\hat{H}$ with eigenvalue 1.

Initially $\hat{B}$ is ~~observed~~ measured and 1 is found. $\therefore$ Afterwards $|\phi\rangle$ is given by $|1\rangle$

$|\phi\rangle$ then evolves as $e^{-\frac{i\hat{H}t}{\hbar}}|\phi\rangle$

Resolve $\quad |\phi\rangle = |1\rangle = x\left(\frac{-2i|1\rangle + |3\rangle}{\sqrt{5}}\right) + y\left(\frac{i|1\rangle + 2|3\rangle}{\sqrt{5}}\right)$

$\therefore \quad x = \frac{2i}{\sqrt{5}}, \quad y = \frac{-i}{\sqrt{5}}$

so $\quad |1\rangle = \frac{2i}{\sqrt{5}}\left(\frac{-2i|1\rangle + |3\rangle}{\sqrt{5}}\right) - \frac{i}{\sqrt{5}}\left(\frac{i|1\rangle + 2|3\rangle}{\sqrt{5}}\right).$