Quantum Mechanics Solutions

42 $i\hbar \frac{d}{dt}|\psi_t\rangle = \hat{H}|\psi_t\rangle$

so if $\hat{H}|E_i\rangle = E_i|E_i\rangle$ are energy eigenstates

and $|\psi_0\rangle = \sum_{i=1}^{\infty} a_i|E_i\rangle$ at $t=0$

$$|\psi_t\rangle = e^{-i\hat{H}t/\hbar}|\psi_0\rangle = \sum_{i=1}^{\infty} a_i e^{-iE_it/\hbar}|E_i\rangle$$

States (normalized) $|1\rangle, |2\rangle, |3\rangle$ satisfy

$$\hat{B}|1\rangle = |1\rangle, \quad \hat{B}|2\rangle = 2|2\rangle, \quad \hat{B}|3\rangle = 3|3\rangle$$

$$\hat{H}|1\rangle = |1\rangle + 2i|3\rangle$$
$$\hat{H}|2\rangle = |2\rangle$$
$$\hat{H}|3\rangle = -2|3\rangle - 2i|1\rangle$$

Clearly $|2\rangle$ is an eigenstate of $\hat{H}$ and the other two are constructed from the linear combinations of $|1\rangle$ & $|3\rangle$

So consider $|\alpha\rangle = A|1\rangle + B|3\rangle$ with eigenvalue E

$$\hat{H}|\alpha\rangle = E|\alpha\rangle = \hat{H}(A|1\rangle + B|3\rangle) = A\hat{H}|1\rangle + B\hat{H}|3\rangle$$
$$= A(|1\rangle + 2i|3\rangle) + B(-2|3\rangle - 2i|1\rangle)$$

so $EA|1\rangle + EB|3\rangle = A(|1\rangle + 2i|3\rangle) + B(-2|3\rangle - 2i|1\rangle)$

$$\therefore \quad EA = A - 2iB$$
$$EB = 2iA - 2B$$