Solitons III: useful integrals

You can quote the following integrals, though deriving them for yourself may be instructive:

- **Indefinite integrals:** [**Note:** the integration constant C can be complex]

$$\int \frac{dx}{x\sqrt{1-x}} = -2\,\mathrm{arcsech}(\sqrt{x}) + C$$

$$\int \frac{dx}{x\sqrt{1-x^2}} = -\mathrm{arcsech}(x) + C$$

$$\int \frac{dx}{x\sqrt{1+x^2}} = -\mathrm{arccosech}(x) + C$$

$$\int \frac{dx}{\sin(x/2)} = 2\log\tan(x/4) + C$$

$$\int \frac{dx}{\cosh(x)} = 2\arctan(e^x) + C$$

$$\int \frac{dx}{1-x^2} = \mathrm{arctanh}(x) + C$$

$$\int dx\,\sqrt{1-x^2} = \frac{1}{2}\left[x\sqrt{1-x^2} + \arcsin(x)\right] + C$$

$$\int \frac{dx}{\cos^2(x)} = \tan(x) + C$$

$$\int \frac{dx}{\cosh^2(x)} = \tanh(x) + C$$

- **Definite integrals:**

$$\int_{-\infty}^{+\infty} dx\,\mathrm{sech}^{2n}(x) = \frac{2^{2n-1}((n-1)!)^2}{(2n-1)!}$$

$$\int_{-\infty}^{+\infty} dx\,e^{-Ax^2} = \sqrt{\frac{\pi}{A}} \qquad (A > 0)$$

Note: the last formula does not change if the integration variable x is shifted by a finite imaginary amount ic, that is if you replace x with $x + ic$. The formula also holds for complex $A = |A|\,e^{i\phi}$ with $-\pi/2 \leq \phi \leq \pi/2$ (so $\mathrm{Re}(A) \geq 0$) and $|A| \neq 0$, provided that the square root of A is defined to be $\sqrt{A} = \sqrt{|A|}\,e^{i\phi/2}$.