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How did I get to be here in Bologna talking to High School Mathematics Pupils & Teachers?

1. Primary: Professor Manaresi's insight to precipitate EU grant;

2. Secondary: Passion for Mathematics, Teaching and the improvement of Matematics in the UK;

3. Coincidence of teaching of module "Mathematics Teaching" to final year Mathematicians.

Talk Overview: What make MT worthwhile for student & employers; Interesting bites; proof.

Maths and Science in the UK are fighting a battle against society & the media — Physics' problem; recollection from a retired student who was a high school inspector.

Mathematics Teaching's aims are unique at Durham.

- To focus on school mathematics from an advanced standpoint
- Reflect on current issues
- Reflect on pupils' learning in secondary schools
- Reflect on students' own mathematical experience
- To develop a fascination for Mathematics

What are the key skills — valued by UK employers:

- **Academic** —- Library research; Synthesis of data; Critical and analytical thinking; Active learning; Problem solving; Project management; Creativity.

- **Self-Management** — Reflective learning; Action planning/Decision-making; Time management/Self-discipline; Independence; Initiative/Proactive approach.

- **Communications** — Written materials; Oral/visual presentations; Active listening; Numeracy; Information skills; Computer skills.

- **Interpersonal** — Group/Teamwork; Understanding/Tolerance of others; Negotiation; Peer assessment; Manage change/Adaptability.

How are they key-skills achieved?

Assessment: 30% Essay; 5% Presentation; 15% School file work; 50% Exam.

School file: 5 visits to secondary school over November. Observe lessons at different levels. Focus on the class learning experience *not* teaching style. Seminars to discuss contrasting school visits.

Interesting Investigations
Every prime number $p \geq 5$ can be expressed in the form $p = \sqrt{24n + 1}$.

At McDonalds in the UK you can get 6, 9 & 20 nuggets* the “McNugget number” is 43, i.e. you can order every number bigger than 43.†

Fun nmemonics — e.g. geometry

Two **O**ld **A**ngels **S**itting **O**n **H**igh **C**hatting **A**bout **H**eaven.

Fibonacci $\{f_n\}_{n=1}^{\infty} = \{1, 1, 2, 3, 5, 8, 13, \cdots\}$ — Cones 8 & 13

$$
\begin{aligned}
1 &= 1 && \text{(\#1)}\\
2 &= 1+1 && \text{(\#1)}\\
3 &= 1+1+1,\ 3 && \text{(\#2)}\\
4 &= 1+1+1+1,\ 3+1,\ 1+3 && \text{(\#3)}\\
5 &= 1+1+1+1+1,\ 3+1+1,\ 1+3+1,\ 1+1+3,\ 5 && \text{(\#5)}
\end{aligned}
$$

*With Happy Meals you get 4: “Mini-McNuggett” number is 11 — Mason & Lomas.

†Contrast with book “Hitchhiker guide to the galaxy.

The number (with $n = 2$)

$$\begin{aligned} x &= 2f_{n+1}f_{n+2} = 2 \times (2 \times 3) = 12 \\ y &= f_n f_{n+3} = 1 \times 5 = 5 \\ z &= (f_{n+1})^2 + (f_{n+2})^2 = 2^2 + 3^2 = 13 \end{aligned}$$

satisft $x^2 + y^2 = z^2$; Lucas sequence start $\{2, 1, \cdots\}$.

Non-standard method of subtraction

$$\begin{array}{r} 437 \\ -249 \\ \hline 118 \end{array} \leftrightarrow \begin{array}{r} 437 \\ +750 \\ \hline 1187 \end{array} \rightarrow 188 \qquad \begin{array}{r} 437 \\ -49 \\ \hline 388 \end{array} \leftrightarrow \begin{array}{r} 437 \\ +950 \\ \hline 1387 \end{array} \rightarrow 388$$

Pick four different digits, then order (descending) as a single number and subtract from the reverse of the number and repeat — eventually you will end up with 6174, e.g.

$$\begin{array}{r} 9532 \\ -2359 \\ \hline 7173 \end{array} \qquad \begin{array}{r} 7731 \\ -1377 \\ \hline 6354 \end{array} \qquad \begin{array}{r} 6543 \\ -3456 \\ \hline 3087 \end{array} \qquad \begin{array}{r} 8730 \\ -0378 \\ \hline 8352 \end{array} \qquad \begin{array}{r} 8532 \\ -2358 \\ \hline 6174 \end{array}$$

Proof

"Most students entering higher education no longer understand that mathematics is a precise discipline in which proof plays an essential role" *Tackling the Mathematics Problem* (1995)

1. Some cautionary examples:

   - Regions of a circle: 1, 2, 4, 8, 16,. . .?
   - Birthday paradox — ≥ 23 in a class then $> 50\%$ of two being on the same day.
   - Numbers of the form $\sqrt{24n+1}$ — $n = 26$.
   - $n^2 - n + 41$ is prime — $n = 41$.

2. Interesting areas “ripe” for proof:

   - Pythagoras Theorem
   - Irrationality of $\sqrt{2}$.
   - The number of primes is infinite.
   - Fermat’s last theorem — no non-zero integer triples solving $x^n + y^n = z^n$ when $n > 2$.
   - Twin primes conjecture: $\{(3,5),\ (5,7),\ (11,13),\cdots??\}$.
   - Mersenne primes (primes of the form $2^n - 1$, e.g. $\{3, 7, 31, 127, 8191, 131071, 524287, \cdots??\}$ — Perfect numbers (number which are the sum of its “factors” $\{6 = 1 + 2 + 3, 28, \cdots, ???\}$.
   - Strong Goldbach conjecture — all positive even integers ≥ 4 can be written as the sum of two primes.

Excellent Mathematics resource: Google — based on a sound mathematical algorithm