LMS Durham Symposium:
Topological Solitons and their Applications
Monday 2nd August - Thursday 12th August 2004

University of Durham, United Kingdom

List of abstracts

Michael Atiyah (University of Edinburgh) Saturday 7th August 09:00

Solitons and Topology

Why are there topological solitons? What about non-topological solitons? How can topology in general help our understanding and study of solitons? These are some of the questions that I will attempt to answer.

Richard Battye (University of Manchester) Wednesday 4th August 16:00

Solitons in multi-component Bose-Einstein Condensates

We will discuss the nature of solitons which can arise in multi-component Bose-Einstein Condensates. In the case of two separate quantum states, it might be possible to create stable vortons and Skyrmions when phase separation takes place.

Roger Bielawski (University of Glasgow) Saturday 7th August 16:30

Cluster decomposition of monopoles and the asymptotic metrics

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Mikhail Bogdan (ILTPE - Ukraine) Thursday 5th August 09:00

Peter Leth Christiansen (Technical University of Denmark) Tuesday 3rd August 09:00

Energy funelling and bubble generation in a bent and twisted DNA model

A plane bent chain of Morse oscillators with long-range dispersive interaction is first considered. Moving localized oscillations may be trapped in the bending region. Thus the chain geometry acts like an impurity. Energy funelling is observed in the case of random initial conditions modelling temperature. Secondly, an augmented model of the DNA molecule including long-range interactions between twisted base pair dipoles is presented. A mechanism for bubble generation is found for sufficiently strong values of the dipole interaction coefficient. The relationship between bubble generation, curvature and twist is investigated. An analytical approach supports the numerical results.

Rossen Dandoloff (Université de Cergy-Pontoise) Monday 9th August 11:30

"Interaction" of solitons and support manifolds

We study classical spin systems which are represented by normalized vector fields. On R^2 the topological configurations are metastable. If the support manifold exhibits intrinsic length, then the solitons have the same characteristic length. If one allows for deformations of the support manifold and the characteristic length of the soliton does not correspond to the characteristic length of the manifold (e.g. the radius of the cylinder), then the soliton may "deform" the manifold in order to "bring closer" the two lengths and minimize its energy. We also study the question if a "deformed" manifold can carry (create) solitons of a field (e.g. Schrodinger) defined on it. This question may be relevant for the energy and charge transport in biopolymers.

Chris Eilbeck (Heriot-Watt University) Tuesday 3rd August 10:00

Breathers on quantum lattices

I discuss the quantum equivalent of a classical breather solution on a discrete lattice. Breathers are solitons-like solutions with an internal degree of freedom. Classical breathers are localized, sometimes mobile, and can be trapped by inhomogenieties or long range forces in the lattice. They display soliton-like approximate stability on collision. I will discuss these effects within the framework of the classical and quantum Discrete Nonlinear Schrödinger model.

Alexander Eremko (Bogolyubov Institute for Theoretical Physics, Kiev) Wednesday 4th August 09:00

Solitons in alpha-helical proteins

(Joint work with L. Brizhik, B. Piette and W. Zakrzewski)

Dynamics of Davydov soliton in an alpha-helical protein macromolecule is studied. It is shown that helical symmetry plays important role in the formation, stability and dynamical properties of solitons. Several types of stationary soliton solutions are found. It is shown that the soliton which spontaneously breaks the local translational and helical symmetries possesses the lowest energy. This soliton has an inner structure which can be described as a modulated multi-hump amplitude distribution of excitations on individual spines. The complex and composite structure of the soliton manifests itself distinctly when the soliton is moving and some interspine oscillations take place. The frequency of these oscillations is shown to be proportional to the soliton velocity.

Two other solitons, which preserve helical symmetry, are shown to be dynamically unstable: once formed, they decay rapidly when they propagate.

Luiz Ferreira (Instituto de Fisica de Sao Carlos IFSC/USP) Wednesday 11th August 17:00

Euclidean 4d exact solitons in a Skyrme type model

We introduce a Skyrme type, four dimensional, Euclidean field theory made of a triplet of scalar fields taking values on the 2-sphere and an additional real scalar field which is dynamical only on a three dimensional surface embedded in R^4. Using a special ansatz we reduce the 4d non-linear equations of motion into linear ordinary differential equations, which lead to the construction of an infinite number of exact soliton solutions with vanishing Euclidean action. The theory possesses a mass scale which fixes the size of the solitons in way which differs from Derrick's scaling arguments.

Alexandre Filippov (BLTP, Joint Institute for Nuclear Research) Wednesday 11th August 11:30

Integrable models of black holes and cosmologies.

Most of the solitons are localized finite solutions of nonlinear two dimensional field theories. Dimensional reductions of gravity or supergravity may lead to two dimensional integrable dilaton gravity theories describing black holes and cosmologies. A rather general class of the models can be reduced to the system of the Liouville equations coupled through two constraints. The general solution of the constraints is given thus providing the general solution of the model in terms of massless free fields. Although the general solution of the Liouville equation is singular there may exist finite solution that, possibly, may be considered as soliton-like objects.

Sergej Flach (Max Planck Institute for the Physics of Complex Systems) Thursday 5th August 10:00

Broken space-time symmetries induced by ac fields: from particle to soliton ratchets

I will explain the symmetry analysis of systems in contact with a heat bath and under the influence of external ac fields. Using the paradigmatic example of a single particle in a periodic potential, I will illustrate the effect of rectification of its motion, discuss the dissipationless limit (Hamiltonian ratchets) and the dynamical mechanisms of rectification. I will then consider spatially extended systems and generalize the symmetry approach to explain rectification of energy currents. The underlying mechanism is intimately related to the presence of topological solitons.

Noah Graham (Middlebury College) Friday 6th August 11:30

Breathers, Q-balls, and Oscillons in Quantum Field Theory

(Joint work with E. Farhi, V. Khemani and R. Markov)

I will begin by reviewing the nonlinear dynmamics of sine-Gordon and kink breathers and Q-balls. Many of the phenomena displayed by these simple examples extend quite generally to broad classes of nonlinear field theories. I will conclude by discussing current work studying applications of these ideas to realistic models of particle physics.

Ruth Gregory (Univeristy of Durham) Tuesday 10th August 16:30

Jarmo Hietarinta (Turku) Wednesday 11th August 16:00

Ribbon knots in the Faddeev-Skyrme model

(Joint work with J Jäykkä and P Salo)

We will discuss vortices and knots in the Faddeev-Skyrme model, and in particular their deformations under dissipative dynamics (i.e., in process towards the minimum energy configuration). The process is illustrated using various methods, including ribbons and their deformations.

Mark Hindmarsh (University of Sussex) Tuesday 10th August 09:00

Dynamical scaling in topological defect networks

Theodora Ioannidou (Aristotle University) Thursday 5th August 09:30

Skyrmions and Harmonic Maps

The rational map ansatz of Houghton et al \cite{HMS} is generalised by allowing the profile function, usually a function of r, to depend also on z and $\bar{z}$. It is shown that, within this ansatz, the energies of the lowest $B=2,3,4$ field configurations of the SU (2) Skyrme model are closer to the corresponding values of the true solutions of the model than those obtained within the original rational map ansatz. In particular, we present plots of the profile functions which do exhibit their dependence on $ z$ and $\bar{z}$. The obvious generalisation of the ansatz to higher SU(N) models involving the introduction of more projectors is briefly mentioned.

Roman Jackiw (MIT) — Tuesday 10th August 10:00

A 100% Cotton Kink in a Funny Place

When the 3-dimensional gravitational Chern-Simons term is reduced to two dimensions, a dilaton-like gravity theory emerges. Its solutions involve kinks, which therefore describe 3-dimensional, conformally flat spaces. Further it is shown that any Poincare-invariant , 2-dimensional equation for a scalar field with dAlembertian kinetic term and arbitrary potential becomes completely integrable when embedded in a 2-dimensional space with curvature determined by the potential.

Marek Karliner (Cambridge and Tel Aviv) — Wednesday 11th August 09:00

Spontaneous breaking of rotational symmetry in many-body and QFT solitons.

I will discuss some examples of spontaneous breaking of rotational symmetry in many-body systems and possible parallels in solitons in quantum field theory.

Stavros Komineas (University of Cambridge) — Monday 9th August 09:00

Solitary waves and vortex rings in a cylindrical Bose-Einstein condensate

Quasi-one-dimensional solitons that may occur in an elongated Bose-Einstein condensate become unstable at high particle density. We study two basic modes of instability and the corresponding bifurcations to genuinely three-dimensional solitary waves such as axisymmetric vortex rings and non-axisymmetric solitonic vortices. We calculate the profiles of the above structures and examine their dependence on the velocity of propagation along a cylindrical trap. At sufficiently high velocity, both the vortex ring and the solitonic vortex transform into an axisymmetric soliton. We also calculate the energy-momentum dispersions and show that a Lieb-type mode appears in the excitation spectrum for all particle densities. We further study numerically head-on collisions between solitons and between vortex rings and show that a shell structure is generically formed during collision. Condensate depletions forming spherical shells have recently been experimentally observed.

Volodya Kopeliovich (Institute for Nuclear Research of RAS) — Wednesday 11th August 10:00

Exotic baryons and multibaryons in topological soliton models

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Arnold Kosevich (B.Verkin Institute for Low Temperature — Saturday 7th August 10:00

Physics and Engineering)

Analogues of dark solitons and Lieb states in a 1D discrete chain

There are several examples of 1D systems where soliton excitations like a dark soliton exist and they are described very well by means of the long wave approximation. Such a description includes even a periodical dependence of the soliton energy on the soliton momentum. The latter allows us to compare this dependence with the dispersion relation for elementary excitations in the system under consideration and find an analogue of such darc solitons with so called Lieb states in 1D Bose gas.

Alexander Kovalev (Institute for Low Temperature Physics and Engineering) Friday 6th August 17:00

Magnetic topological excitations in nanodots and simple models

Domain wall in ferromagnet was historically the first 1D topological object which was investigated both experimentally and theoretically in solid state physics. In 2D magnets the family of nonlinear excitations is much more vast. Domain walls and cilindrical domain walls, magnetic solitons, rotary waves, skirmions and vortices, targets and spirals represent the examples of such excitations. Vortex excitations in this list take on special significance because (1) they control the transition into magnetically ordered state; (2) are experimentally observed and (3) maybe will play an important role in noval electronic equipments. Now one of the most interesting subjects in physics of magnetism is connected with magnetic nanodots and artificial lattices of such magnetic dots. Predicted many years ago magnetic vortex state has recently received renewed attention since it is often found to be the ground state of nanodots. This is why the theoretical investigation of the structure and dynamics of the vortices in small particles is now very important. Many results in this area were obtained by numerical simulations. (The standard size of systems in these simulations is of order of 100 sites, i.e. of order of real nanodots with the diameter ~500A). Some problems of vortex dynamics may be solved analytically in the framework of models with a finite number of degrees of freedom. (These models are related to so called "magnetic molecules" as well). In particular the classification of internal modes of a vortex, the switching of vortex polarity in a circular external magnetic field and an influence of magnetic dipol interaction on vortex structure were investigated in particles of small size. Another kind of vortex dynamics - its motion may be analytically studied in one - dimensional model in which a discrete and continuous approaches are combined. In all above - named problems the important and principal questions related to the features of topology in discrete and finite size systems arise.

George Lavrelashvili (A. Razmadze Mathematical Institute, Tbilisi) Tuesday 10th August 11:30

Non-Abelian gravitational solitons

I plan to review properties of the Bartnik-McKinnon solutions and their generalizations with the dilaton and a cosmological constant.

Nick Manton (University of Cambridge) Friday 6th August 10:00

The Kähler potential for Abelian Higgs vortices

(Joint work with H-Y Chen)

The Kähler potential on the moduli space of vortices is calculated in two different ways, one involving a scaling argument, and the other using an analogue of the regularized action of

Liouville field theory.

Franz Mertens (University of Bayreuth)

Thursday 5th August
11:30

Effects of ac magnetic fields on the vortex dynamics in 2D magnets

In two-dimensional anisotropic classical Heisenberg models with XY symmetry there are non-planar vortices which exhibit a localized structure of the z-components of the spins around the vortex center. We study how an in-plane ac magnetic field induces a switching of this structure into the opposite direction. In our simulations of the many-spin system the vortex goes to a circular limit trajectory, with an orbit frequency which is lower than the driving frequency. We develop a collective variable theory which accounts for the internal degrees of freedom of the vortex core. The evolution equations for the collective variables yield limit-cycle solutions in qualitative agreement with the simulation results.

Sergei Mingaleev (Universität Karhlruhe)

Thursday 5th August
16:30

Nonlinear photonic crystals: a new type of soliton-bearing systems

In this talk, I shall briefly overview the main properties of nonlinear two-dimensional photonic crystals and shall show that they provide us with a new type of soliton-bearing systems which can be described by a system of coupled NLS equations with essentially long-range inter-site coupling. I shall discuss properties of different types of possible nonlinear excitations in such systems with the emphasize on the possibility of their experimental observation.

Franco Musumeci (DMFCI - Catania University)

Wednesday 4th August
11:30

Relation between delayed luminescence and structures of simple biological systems

New experimental results about the Delayed Luminescence (DL), emitted by simple living organisms and biological tissues, after illumination, are described. Some evidences of the connection between this phenomenon and the functional state of the biological systems are reported. A possible origin of DL as due to collective states exited in low-dimensional biological macromolecules, like alpha-helical proteins, actin filaments, microtubules, is suggested.

Antti Niemi (Uppsala Universitet)

Wednesday 4th August
16:30

Nicos Papanicolaou (University of Crete)

Friday 6th August 09:00

Commensurate and incommensurate magnetism in layered antiferromagnets

Recent experiments have shown that the magnetic compound Ba_2CuGe_2O_7 is an essentially two-dimensional antiferromagnet that exhibits an incommensurate-to-commensurate phase transition when an applied magnetic field exceeds a certain critical vale H0, in analogy with the field induced cholesteric-to-nematic transition observed in chiral liquid crystals. The low-energy dynamics is described here in terms of a continuum field theory in the form of a nonlinear sigma model. We are thus in a position to carry out a complete calculation of the ground state and the corresponding magnon spectrum for any strenght of the applied field throughout the phase transition. In particular, our spin-wave analysis reveals field induced instabilities at two distinct critical fields H1 and H2 such that H1 < H0 < H2. Hence we predict the existence of an intermediate phase characterized by a nutating conical spiral. Analogous considerations apply to the layered antiferromagnets Ba_2CoGe_2O_7 and K_2V_3O_8.

Markus Quandt (Universität Tübingen) TBA

Center Vortices in Yang-Mills Theory

Center vortices are extended strings of quantised magnetic flux which play a dominant role in the low-energy sector of non-Abelian gauge theories. After reviewing some formal aspects of vortices I present lattice evidence for the random vortex scenario of quark confinement. The equivalent of lattice vortices can also be defined in the continuum, which allows to study their topology as well as the effective vortex interactions. Based on these findings, I present an effective model of random vortex surfaces aimed at describing the infrared sector of Yang-Mills theory. This model reproduces many properties of non-perturbative gauge theory, in particular the order of the deconfinement phase transtition at high temperatures and the interface tension involved. Furthermore, the geometrical structure of center vortices is studied, including vortex branchings, which are a novel property of the gauge group G=SU(3). Finally, I present preliminary results for the 't Hooft loop operator which is a dual order parameter for the deconfinement phase transition.

Markus Quandt (Universität Tübingen) Monday 9th August 10:00

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Arttu Rajantie (University of Cambridge) Friday 6th August 16:00

Quantum solitons and dualities on lattice

I explain how lattice Monte Carlo simulations can be used to study topological solitons in quantum gauge field theories in a fully nonperturbative way. I present accurate measurements of the vortex mass in the 2+1D Abelian Higgs model, and show that they support a conjectured duality of that model with a scalar field theory. I also present preliminary results for the mass of the 't Hooft-Polyakov monopole in the 3+1D Georgi-Glashow model and discuss the possibility of a similar dual description.

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Avadh Saxena (Los Alamos National Laboratory) Saturday 7th August 11:30

Elasticity of Membranes and Vesicles: Role of Topology

(Joint work with Jerome Benoit (Univ. Crete, Greece) and Turab Lookman (LANL))

The shape change of closed membranes (vesicles) is important for biological function and can be understood within the framework of bending elasticity. Apart from the usual spherical vesicles, toroidal vesicles of higher genus have also been observed. Clearly, an interplay of curvature and topology is an essential feature for understanding the deformation of vesicles. We apply the Bogomol'nyi technique, which is usually invoked in the study of solitons or models with topological invariants, to the case of elastic energy of vesicles. We show that spontaneous bending contribution caused by any deformation from metastable bending shapes falls in two distinct topological sets: shapes of spherical topology and shapes of non-spherical topology experience respectively a deviatoric bending contribution and a mean curvature bending contribution. We calculate the elastic energy as a function of genus. We also present an exact solution to the problem of the global shape description of a spherical vesicle distorted by a grafted latex bead.

Bernd Schroers (Heriot-Watt University) Saturday 7th August 16:00

Electric-magnetic symmetry and confinement in discrete gauge theories

Gauge theories where the gauge group is spontaneously broken to a discrete group have two types of excitations: topological vortex solutions carrying discrete magnetic flux and perturbative excitations carrying electric charge. All excitations can be organised into irreducible representations of a symmetry algebra which includes the electric and magnetic symmetry. In this setting one can show explicitly how breaking of electric symmetry leads to magnetic confinement and vice-versa, thus realising the 't Hooft-Mandelstam picture of confinement in a simpler but mathematically precise setting.

Martin Speight (University of Leeds) Tuesday 10th August 16:00

Algebraic topology of the Skyrme configuration space

(Joint work with Dave Auckly, Kansas State)

I will explain how to compute the fundamental group of the space of mappings from an oriented 3-manifold to an arbitrary Lie group, and use the result to study the quantization of exotic Skyrmions.

Paul Sutcliffe (University of Kent) Tuesday 3rd August 16:00

Skyrmions and the pion mass

Nikos Theodorakopoulos (Theor. & Phys. Chem. Inst., NHRF, Athens) Wednesday 4th August 10:00

Discrete topological solitons and one-dimensional thermodynamic instabilities: application to DNA melting and unzipping

The mechanical or thermal unbinding of the two DNA strands, commonly known as "unzipping", or "thermal melting", respectively, can be described as a one-dimensional instability. This is a true thermodynamic transition not prohibited by Landau's theorem, because the nonlinear structure (topological soliton, domain wall (DW)) which interpolates between the bound and unbound phases has a macroscopically large energy (proportional to the number of unbound sites).

The detailed properties of the DW's will be derived in the context of the discrete Peyrard-Bishop Hamiltonian. It will be shown that at high values of the discretization parameter (i.e., near the anticontinuum limit), a multitude of DW-like structures is possible for a given length of the unbound segment. The stability properties of this family of DW's will be discussed. The DW thermodynamics will be formulated, both in the Gaussian approximation and in lowest-order anharmonic perturbation theory. Results for unzipping and thermal melting will be compared with those obtained by other numerical methods.

Jack Tuszynski (University of Alberta) Monday 9th August 16:00

The role of nonlinearities in the biophysical properties and biological function of key subcellular components

This talk discusses the role of nonlinearities in the physical description of several key biomolecules that participate in a number of crucial subcellular processes, namely actin, microtubules and ions crowding around these filaments. We show that the assembly kinetics of actin is a nonlinear process that requires not only a mechanism of saturation but also annealing and fragmentation that are governed by coupled nonlinear equations involving monomer concentration and filament number as the key dynamical variables. The observed dendritic growth of actin networks in cell motility phenomena is subsequently described by the coupling of actin filaments to the protein called Arp 2/3. We then investigate the role of nonlinear dynamics in the formation of microtubules. First of all, space-flight laboratory experiments have shown that the in vitro and in vivo self-organization of microtubules is sensitive to gravitational conditions. We propose a model of self-organization of microtubules in a gravitational field. The model is based on the dominant chemical kinetics. The pattern formation of microtubule concentration is obtained: 1) in terms of a moving kink in the limit when the disassembly rate is negligible, and 2) for the case of no free tubulin and only assembled microtubules present. The results of our simulations are in good quantitative agreement with experimental data. Next, we present a recently proposed model of molecular and bulk elastic properties of microtubules that include macroscopic estimates of the anisotropic elastic moduli of microtubules, accounting for the molecular forces between tubulin dimers: for a longitudinal compression, for a lateral force and for a shearing force. At the level of large bending motions of microtubule filaments, a continuous medium model is proposed describing a microtubule as an elastic rod. Keeping the dominant nonlinear terms in the bending dynamics equation, we found that when the microtubule is subjected to bending forces, the deviation angle satisfies a Sine-Gordon equation. Particular analytical solutions of this equation are found which describe kink and anti-kink bending modes that may propagate at a range of velocities along the length of the microtubule. Kinetic energies and characteristic damping times of these modes are calculated for different propagation velocities and compared with thermal and ATP hydrolysis energies. Finally, we discuss how coupled differential equations describing the interactions between ions in solution and the filament they surround can lead to solitonic signal transmission.

Niels Walet (UMIST) Tuesday 3rd August 11:30

Nonlinear dynamics of electrons in chiral molecules

We present a very simple model for the study of charge transport in a molecule patterned on B-DNA. In this model we use a discrete non-linear Schrödinger equation to describe electrons propagating along the sugar-phosphate backbone of the DNA molecule. We find that in this model, for a given nonlinearity, the transport is controlled by J, a parameter which relates to the electronic coupling between different molecules on the backbone. For smaller values of J we have localised states while at higher values of J the soliton field is spread out and through its interaction with the lattice it has stronger effects on the distortion of the lattice.

Yaroslav Zolotaryuk (Bogolyubov Institute for Theoretical Physics, Kiev) Tuesday 3rd August 17:00

Mobility of topological solitons in the discrete nonlinear Klein-Gordon models

We study the mobility of topological solitonic excitations in the discrete Klein-Gordon equation. We demonstrate that, depending on the shape of the on-site potential, a discrete set of velocities can appear, for which an (anti)kink can propagate freely. On several examles we show that this is a generic effect which takes place for potentials with rather flat barriers and narrow wells.

[1] A.V.~Savin, Y.~Zolotaryuk and J.C.~Eilbeck, Physica D, v. 138, pp. 265--279, 2000.

[2] S.~Flach, Y.~Zolotaryuk, and K.~Kladko, Phys. Rev. E, v. 59, pp. 6105--6115, 1999.