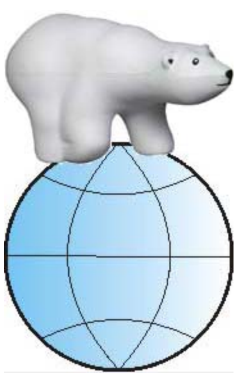
Distant Mathematics Challenge 2011

sponsored by Department of Mathematical Sciences, Durham University

Aim. The challenge is designed to help select students for the Gifted and Talented Summer School at Durham University in August 2011. For full details of the application process, please contact Mr Shane Collins (s.m.collins@dur.ac.uk). Our wider aim is to promote mathematical thinking. Submissions from all students in year groups 7–13 are welcome.

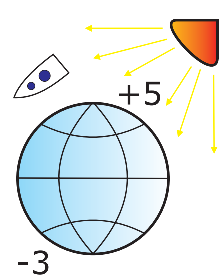
Please post **complete solutions** to Dr Vitaliy Kurlin (Department of Mathematical Sciences, Durham University, Durham DH1 3LE) by 5pm on Friday 1st April 2011. If you have any questions on the problems, please e-mail vitaliy.kurlin@durham.ac.uk. You may also e-mail your solutions, simply scan your script or use a photo camera. On page 1, please write your age group, your school name and the name of your mathematics teacher.

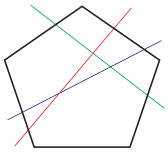
Marking. The credit is given only for justified mathematical arguments. The full mark for each of eight problems is 10. Marks for each problem will be rescaled according to the actual difficulty of the problem (the average mark over all submissions). The results of the challenge will be available in April 2011 at http://www.maths.dur.ac.uk/∼dma0vk/challenge.html.



Problem 1: *Attenborough and a bear.*
David Attenborough walked two miles from his camp in the eastward direction. Then he turned to the north and walked one mile. Suddenly, he spotted a bear and started to run to the south. After running one mile to the south, David reached the camp. What was the colour of the bear?

Problem 2: *the first spaceship.*
50 years ago the first manned spaceship made a few turns (90 min each) around Earth. During the first half of each turn on the sunny side of Earth, the spaceship was getting 5 degrees warmer. During the second half of each turn, the spaceship was becoming 3 degrees cooler. How long into the flight will the original temperature increase by 35 degrees?

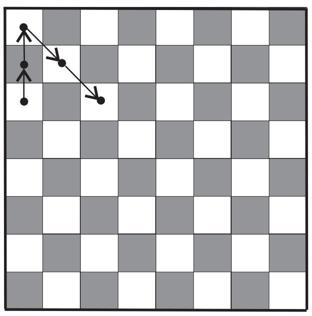




Problem 3: *share a cake with many friends.*
You are cutting a pentagonal cake along straight lines. What is the maximum number of pieces that you can get after 99 cuts?

Problem 4: *count a lot of sequences.*
An ordered sequence consists of only 1 or 2. The sum of all elements is 15. How many such sequences are there?

the sum is 4:
1111, 112,
121, 211, 22

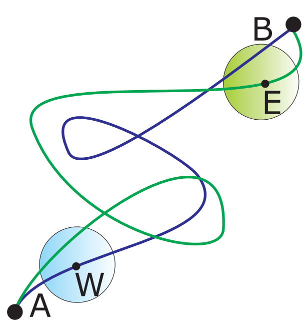


Problem 5: *a long way for a queen.*
Walking on a chessboard 8×8, a queen passed through the centre of each cell once. The path of the queen turned out to be closed and without self-intersections. How short and how long can this path be? All cells are unit squares. A queen can move in any direction: horizontally, vertically, or along a diagonal.

Problem 6: *a simple multiple of 2011.*
Find a number consisting of only 1s that is divisible by 2011.

Best Wishes for the Year 2011!

Problem 7: *cockroaches and robots.*
Two tiny cockroaches were tied to each other by a thread of length 20 cm.



Despite this constraint, the cockroaches managed to run from one corner A to the opposite corner B along quite arbitrary paths. Robotic cleaners W and E are round disks of radius 10cm. The robot W is moving from A to B along the path of the first cockroach. The robot E is moving from B to A along the path of the second cockroach. Prove that the robots will inevitably collide in all cases.

The cockroaches and the robots can choose any changeable velocities.

Problem 8: *pick the right apples!* There are 2011 apples on several trees: fewer than 262 apples on each tree. Can you pick apples in such a way that (1) there are at least 262 apples left, and (2) all the trees that still have some apples have exactly the same number of apples?