Riemannian Geometry IV

Problems, set 7.

Exercise 18. Let $H^n = \{(x_1, \ldots, x_n) \in \mathbb{R}^n \mid x_n > 0\}$ denote the upper half space model of the hyperbolic space with its standard Riemannian metric

$$g_x(v_1, v_2) = \frac{\langle v_1, v_2 \rangle_0}{x_n^2},$$

where $v_1, v_2 \in T_xH^n \cong \mathbb{R}^n$ and $\langle \cdot, \cdot \rangle_0$ denotes the standard Euclidean inner product. Calculate all the Christoffel symbols $\Gamma^k_{ij}$ with respect to the canonical global coordinate chart $\varphi : H^n \to V \subset \mathbb{R}^n$, $\varphi(x) = x$. 

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