1. (a) We have
\[ g^{(k)}(x) = \begin{cases} 
0, & x \leq 0, \\
\frac{p(x)}{q(x)}e^{-\frac{1}{x^2}}, & x > 0,
\end{cases} \]
with appropriate polynomials \( p, q \). Obviously,
\[ \lim_{x \to 0} \frac{p(x)}{q(x)}e^{-\frac{1}{x^2}} = 0, \]
so \( g \) is a smooth function. Similarly, \( h(x) \) is a smooth function, and so is the product \( g(x-a)b(x-b) \). Since \( g(x-a) = 0 \) for all \( x \leq a \) and \( f(x-b) = 0 \) for all \( x \geq b \), we have \( f_{a,b}(x) = 0 \) for \( x \leq a \) and \( x \geq b \). Note that for \( x \in (a, b) \) we have
\[ f_{a,b} = e^{-\frac{1}{(x-a)^2}}e^{-\frac{1}{(x-b)^2}} > 0. \]
This proves (a).

(b) Note that for every \( x \in [-10, 10] \), there is at least one function \( F_i(x) > 0 \). Hence, we have \( F_1(x) + F_2(x) + F_3(x) > 0 \) for all \( x \in [-10, 10] \). Obviously, we have
\[ f_1(x) + f_2(x) + f_3(x) = \frac{F_1(x) + F_2(x) + F_3(x)}{F_1(x) + F_2(x) + F_3(x)} = 1, \]
and all functions \( f_i : [-10, 10] \to [0, 1] \) are smooth. One easily checks with (a) that their supports lie in \( U_1, U_2 \) and \( U_3 \).

2. \( K \) is a manifold with boundary, and if we choose the identity map as global parametrisation, the induced orientation on its boundary \( E = \partial K \) is such that the outer normal unit vector field of \( K \) is positively oriented. By Stokes’ Theorem, we then have
\[ \int_K d\omega = \int_E \omega = 4\pi abc. \]
On the other hand, we have
\[ \int_K d\omega = 3 \int_K dx \wedge dy \wedge dz = 3 \int_K 1dx \, dy \, dz = 3\text{vol}(K). \]
Putting both results together, we end up with
\[ \text{vol}(K) = \frac{4}{3} \pi abc. \]