Maths Outside – taking maths out of the classroom.

Learning mathematics outside the classroom is not enrichment, it is at the core of empowering an individuals understanding of the subject.

Mathematics educators have suggested that students should receive opportunities to use and apply mathematics and engage in mathematical modelling (1). Constructivists state that students do not merely learn by being told a set of instructions to follow and that they would benefit from opportunities to model maths in real world situations. Jo Boaler (2) states that

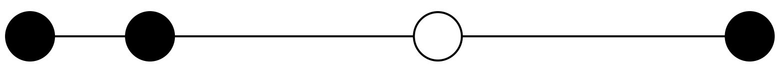
Students do not only learn knowledge in the mathematics classrooms, they learn a set of practices and these come to define their knowledge…students that engage in mathematical modelling can encourage them to develop a deeper, conceptual knowledge of mathematics…that can have value beyond the classroom.

The three activities described here are some of the many maths activities which can be used outside the classroom. Delivering mathematical concepts in this way engages and reinforces learning. It puts the ideas learnt into a setting and allows time for those ideas to be developed without any of the maths hang-ups which can occur in the classroom. By taking maths beyond the classroom, we can more clearly illustrate the connections between the real world and what is being studied in school. In so doing we are enthused by the wealth of resources all around us in the environment. [3]

A length of rope is a great prop to start. Mix this up with a magical constant called The Rendezvous Constant [4], a name which suggests feelings of adventure before you even start to understand its origins.
Mark a number line with a piece of chalk from 0 to 10, in 30cm divisions, on the ground. Ask three people to stand at any point on the line. A fourth volunteer then stands at another point. The sum of the three distances from this point to the other points has to be 15 if it is the Rendezvous point. The Rendezvous constant for a straight line is the midpoint, in this case 5.
A general definition for the Rendezvous Constant is given by Gross's Theorem:
For **any** collection of points $x_1, x_2, ..., x_n \in E$, there is a point $y \in E$ for which the average distance from y to $x_1, x_2, ..., x_n$ is $a(E)$, i.e.

$$\frac{1}{n}\sum_{i=1}^{n}|x_i - y| = a(E) \qquad - (1)$$

We emphasize that, although y varies with the collection of points $x_1, x_2, ..., x_n$, the Rendezvous constant, $a(E)$, works for all collections of points and no other constant works.



Next use a straight line of length 10, with three counters placed at $x_1 = 0$, $x_2 = 1$, $x_3 = 10$ and $n = 3$. See above.

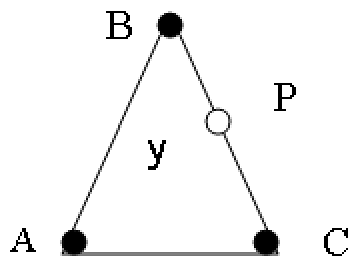
Equation (1) then gives: $\frac{1}{3}[(y-0)+(y-1)+(10-y)] = a(E)$

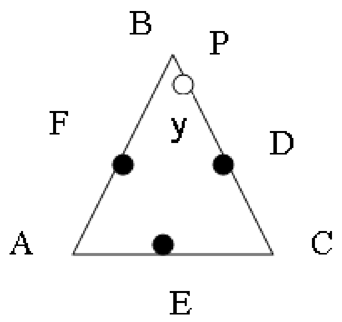
which with $a(E) = 5$ gives $y = 6$, placed at the white circle.

The best way to find Rendezvous Constants is to initially experiment with the help of a rope. The rope for a 300cm number line 0 to 10 (10x30cm divisions), has to be 5x3x(2x30cm)=900cm. Making the rope twice as long as is required enables you to start at the Rendezvous point (y) person and go out to one of the three people on the line, and then back to the y point, repeating this for all three people. If there is any surplus/shortage of rope when this is completed it will give a clue to the group as to what is needed to be done to get the correct point. Repeating this practical experiment reinforces the concepts. Then you are then ready to start and move to algebra to verify the approximate solutions exactly, some of which will involve looking at modulus linear equations.

There are Rendezvous Constants for all shapes. Regular polygons offer some wonderful opportunities to work on experiments and the mathematics of proof.

Eg. The perimeter of an equilateral triangle with sides of length 1 can be shown to have a Rendezvous Constant of $\frac{2+\sqrt{3}}{6}$. This is a 1-dimensional case, as the points are constrained to lie on the perimeter of the triangle.





Firstly let $n = 3$, with the three points x_1, x_2, x_3 being the vertices A,B,C of the triangle in Figure 2a. Then $\frac{1}{3}\sum_{i=1}^{3}|x_i - y| = \frac{1}{3}(|BP|+|PC|+|AP|) = \frac{1}{3}(1+|AP|)$. This is a minimum when P is the midpoint of BC, so that $a(E) \geq \frac{2+\sqrt{3}}{6}$

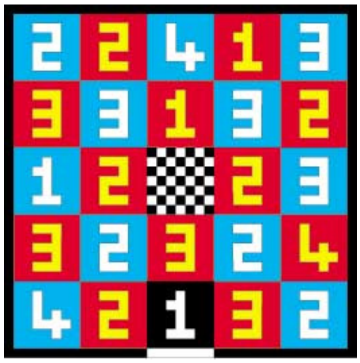
Secondly, let $n = 3$, with the three points x_1, x_2, x_3 being the midpoints D,E,F of each side. See Figure 2b.

Then $\frac{1}{3}\sum_{i=1}^{3}|x_i - y| = \frac{1}{3}(|PE| + |PD| + |PF|) \leq \frac{1}{3}(|PE| + |PD| + |PB| + |BF|) = \frac{1}{3}(1 + |PE|)$.

This is maximum when y is at vertex B, so that $a(E) \leq \frac{2+\sqrt{3}}{6}$. Together this gives the unique Rendezvous Constant as $a(E) = \frac{2+\sqrt{3}}{6}$.

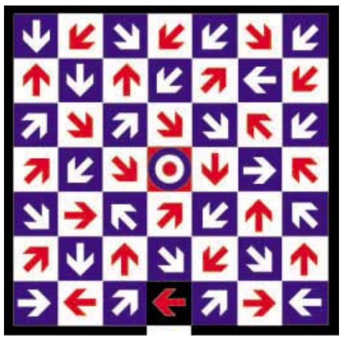
The same method can be used to find the Rendezvous Constant for the perimeter of all regular polygons

Adrian's Fishers 6 minute number and arrow mazes offer opportunities to deal with mathematical proof. [5]



Jumping Maze

Start at the black square with the "1" on it in the centre of the bottom row, and jump forward, sideways, or backwards, but never diagonally, the number of squares indicated by the number on the square. The objective is to find your way to the central square.

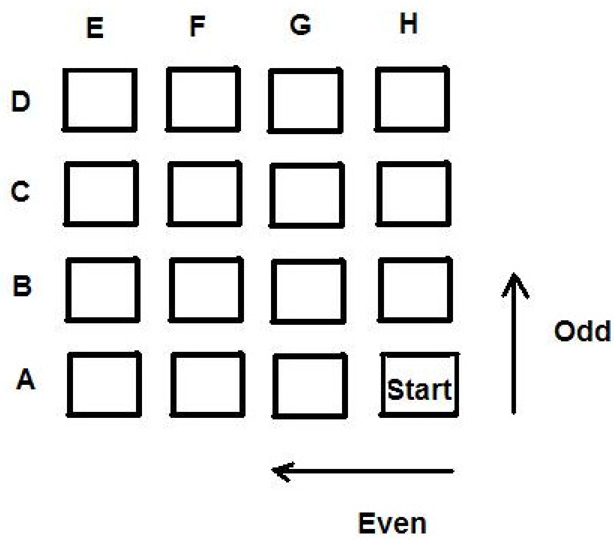


Archery Maze

Starting at the arrow in the black square, find your way to the central target. Move any distance in the direction indicated. Whenever you stop, chance direction as indicated by the arrow on which you land

Once students have an understanding of how to solve these mazes they can then create their own mazes. Asking questions about the number of routes to solve the maze and if it is possible to create a maze with a unique solution. Group can then chalks out their own grids and challenge others to solve their mazes.

Grid Catchers [6] is a game played with a four sided dice! In other words you could use a normal dice and throw again if you get five or six. Alternatively use a spinner numbered one to four. Starting in the bottom right hand corner throw a dice if you get an odd number move up, even number move left the value of the score, with the aim to get off the grid. Before the game starts everyone else has to make a decision about which point you will exit the grid. The catchers have to pick A to H marked on the diagram below and stand by this point to catch. Variations on the game involve the following ideas;



a) Most likely exit point.
b) Which points will you never come off the grid
c) Least likely exit points (other than (b))
d) If you had to pick 3 places to stand to catch, which would you pick?
e) Calculate the probabilities of where you will come off the grid

These are just a few of the many activities which can be used outside. Others are such as

A) Puzzles walks can also be used to “open peoples mathematical eyes” and create awareness of the maths that is all around them in number, shape, architecture and design [7]

B) Mathematical Card Magic such as Kruskal’s Card Count and other such tricks can be played with giant cards on the pavement surface [8]

C) Giant blocks can be used to balance out to infinity. [9]

D) Art maths can be used to link to design, fashion and architecture. [10]

References

[1] A. Schoenfeld, Learning to think mathematically: problem solving, metacognition, and sence making in mathematics, Handbook of Research on Mathematics Teaching nad Learning, pp 334-371,New York: MacMillan 1992
[1] W. Blum & M. Niss, Applied mathematical problem solving, modelling, applications and links to other subjects, Educational Studies in Mathematics, 11, pp37-69, 1991
[2] J Boaler, Mathematical modelling and New theories of learning, Teaching Mathematics and its Applications, 20, no. 3, 2001
[3] S.Humble, The Experimenter's A to Z of Mathematics, David Fulton, 2002
[4] S.Humble, Rendezvous Constants, Mathematical Gazette July 2008
[5] 6 minute mazes www.adrianfisherdesign.com/live_site/
[6] S. Humble, Catch Me if you can! TMME, vol6, no1and 2
[7] Dr Maths Puzzle trails www.ima.org.uk/Education/DrMaths/DrMaths.htm
[8] S. Humble, Magic Cards TMME vol 5, no 2
[9] Y. Nishiyama. Building Blocks Problem Related to Harmonic Series, TMME, vol 3, no 1
[10] Learning Maths Outside the Classroom
www.ncetm.org.uk/resources/9268

Steve Humble

Steve Humble (aka Dr Maths) has developed the Learning Maths Outside the Classroom mini-site on the www.ncetm.org.uk website. As Dr Maths he has organised a number of street maths busking events such as "Maths on the Quayside". 2600 school children took part in this maths trail around Newcastle and Gateshead Quayside in June 2007. He holds the Guinness World Record for the Largest Maths Class Outside. He is the Chair of the organising committee for the June 2010 conference How to Talk Maths in Public.
For more information on Dr Maths go to
www.ima.org.uk/Education/DrMaths/DrMaths.htm
email drmaths@hotmail.co.uk