Mathematics Carousel

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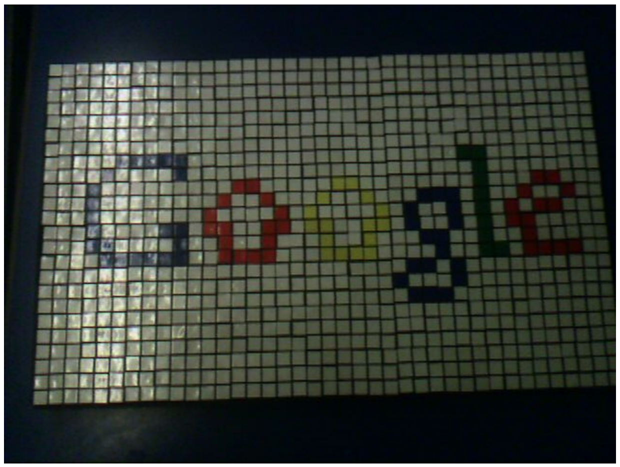
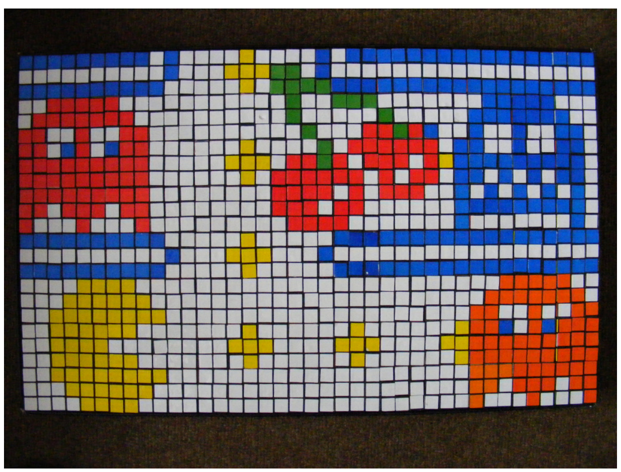
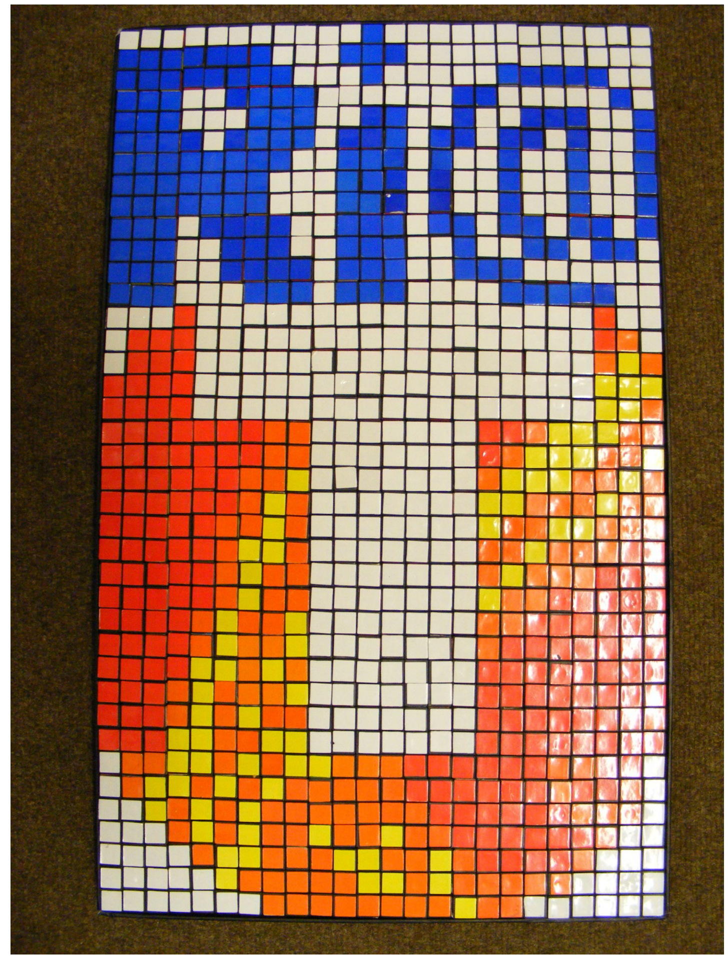
Some FMSP cubist pictures







Overview

- Platonic solids - Cubism
- Tilings
- Steiner networks
- Proof
- Bell-ringing by Joyce Brown

What makes a good mathematician? nrich.maths.org

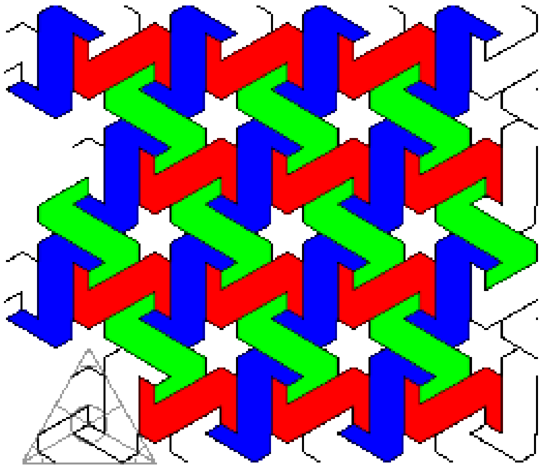
- Logical thinking; Systematic.
- Intuitive; Creative.
- Practice; Dedication; Obsessive.
- Enthusiasm; Love for the subject.
- Solve problems.
- Rigourous – “stonebreakers who grind to dust”.

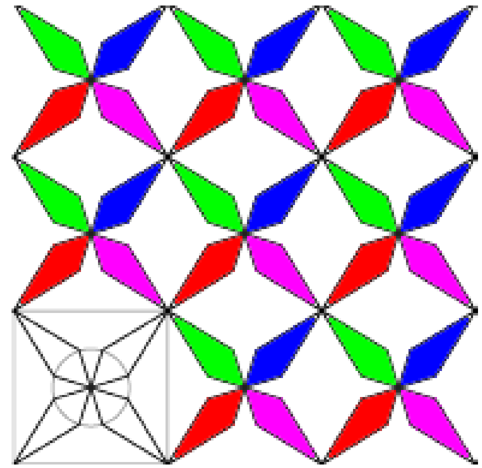
A Platonic Solid is a solid bounded by plane faces where each face is the same.

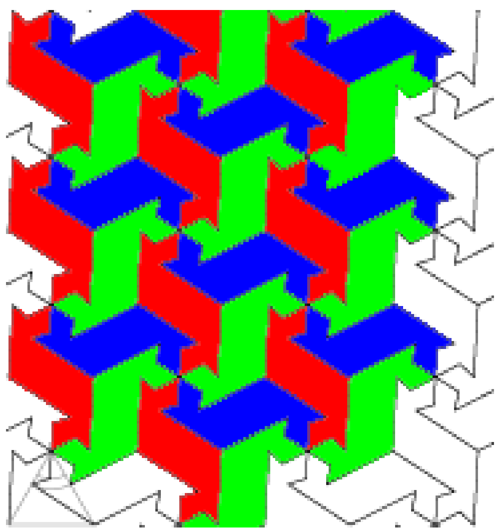
How many Platonic solids are there?

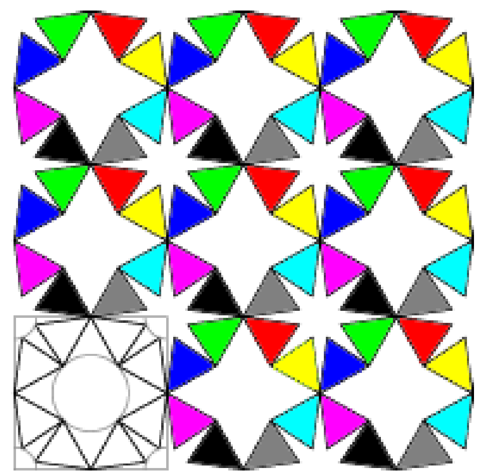
Give an overview of some twisty puzzles and what makes them “special”

Regular tilings for colouring









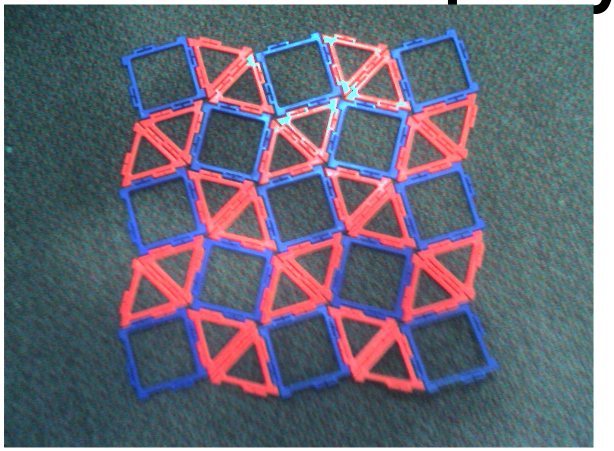
An aside: Colour

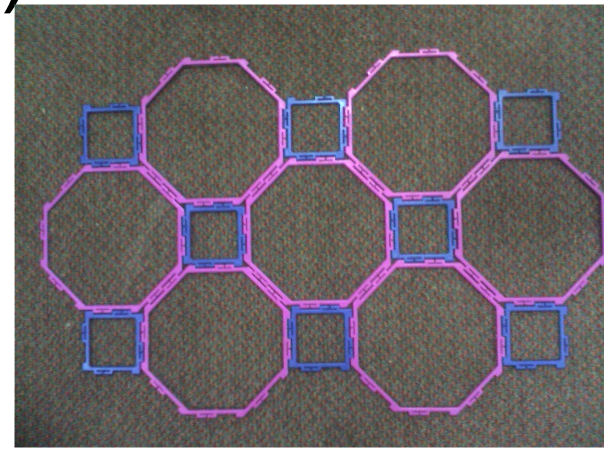
- Each cell has a unique colour made up by mixing Red/Green/Blue which can be represented as (R,G,B).
- In html (webpages) colours are represented as #1B3D5F where pairs of digits are in hexadecimal which is the industry standard – actually the custom colour is shown!

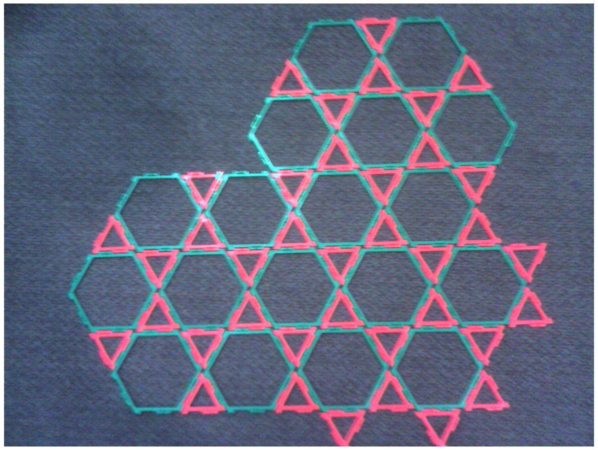
Postscript language

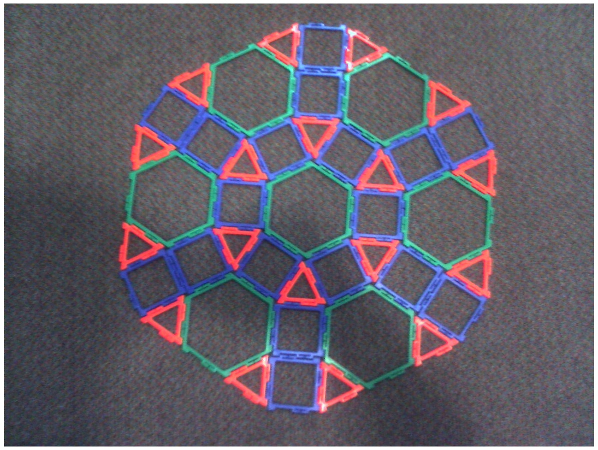
- PostScript is best known for its use as a page description language in the electronic and desktop publishing areas – its child is PDF.
- You define objects and the language is mathematical. There are commands for “scale enlargement”, “translate”, see an example.

Irregular tilings (mix regular polygons)









A systematic approach leads to:

- Arriving at the formula for an internal angle of a regular n-gon: (n-2)*180/n

Name	Number of sides	Internal angle (degrees)
Triangle	3	60
Square	4	90
Pentagon	5	108
Hexagon	6	120
Octagon	8	135
Nonagon	9	140
Decagon	10	144
Dodecagon	12	150

- Only n-gons where n is a factor of 180 is possible.
- Take a few minutes to find some answers which could lead to tessellation

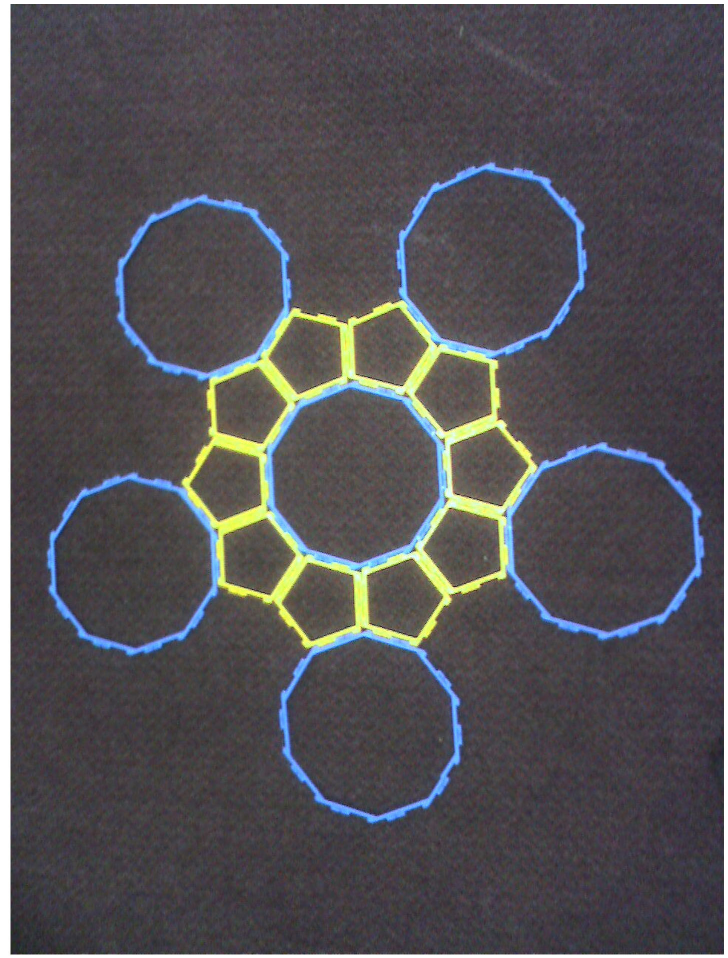
- Answers diophantine equations:

Some answers:

360=150+90+2x60
360=144+2x108
360=2x135+90
360=3x120
360=2x120+2x60
360=120+4x60
360=120+2x90+60
360=4x90
360=2x90+3x60
360=6x60

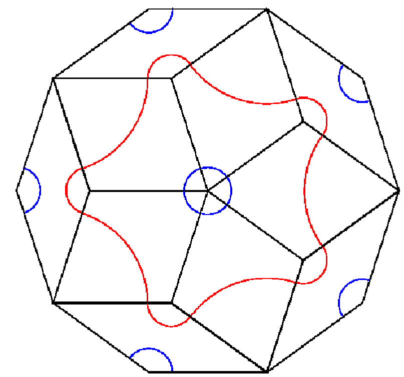
Counter-example

- 360=144+2x108

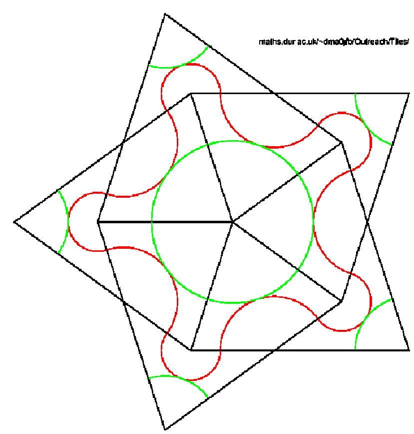


Penrose tilings

- Two Penrose tilings



maths.dur.ac.uk/~dma0jfb/Outreach/Tiles/



A mathematical game

Using 6 >= numbers >= 1, how many ways can you write 8 using exactly six numbers?

- Two ways:

8=1+1+1+1+1+3

8=1+1+1+1+2+2

If we used these numbers as a six-sided dice with a sum total of 8 which would be best?

	1	1	1	1	1	3
1	D	D	D	D	D	L
1	D	D	D	D	D	L
1	D	D	D	D	D	L
1	D	D	D	D	D	L
2	W	W	W	W	W	L
2	W	W	W	W	W	L

Probability purple wins = 10/36,
Probability purple loses= 6/36.

What if we increase the sum total – use Maple.

Group Activity

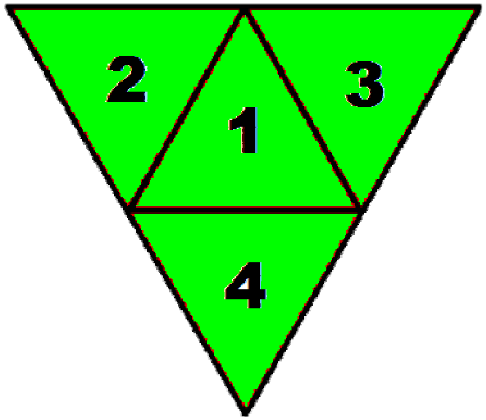
- Each student should take a blank dice and write a number on each side according to the following two rules:
  - You can only use the numbers 1, 2, 3, 4, 5, 6.
  - The total of the six numbers number be 21.
- Form into eight groups of four and play a knock-out competition for the best of ten game:

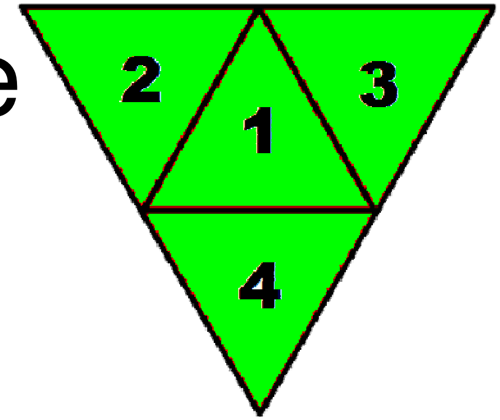
Amongst the group of four can you work out the best dice?

- There is a best/worst dice – yes up to n=15.
- The first set of non-transitive dice is when n=14 in which case 4 beats 6, 6 beats 11 and 11 beats 4.
- For n=21 the “best” dice are
  [1, 1, 3, 5, 5, 6]
  [1, 3, 3, 4, 5, 5]

Sicherman Dice

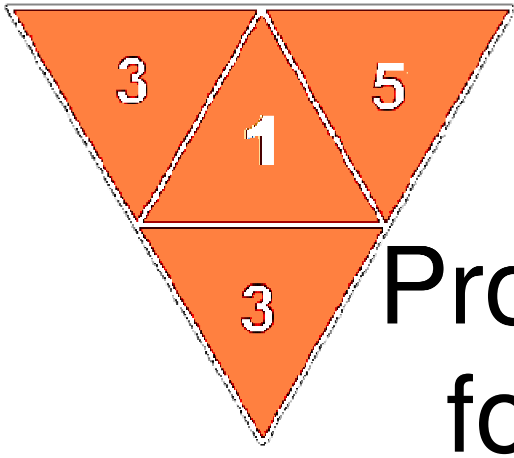
Probability distribution for Two Standard Dice



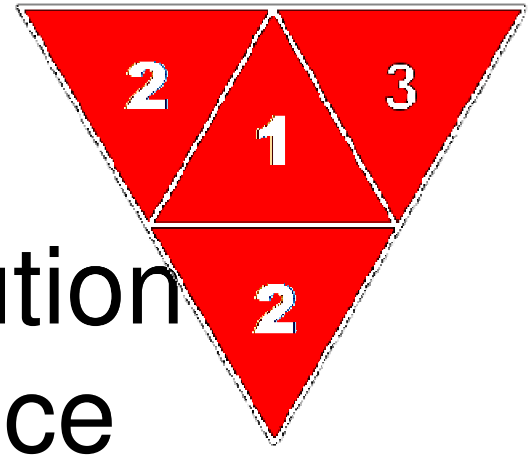


	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

P(2)=1/16, P(3)=2/16, P(4)=3/16, P(5)=4/16, P(6)=3/16, P(7)=2/16, P(8)=1/16



Probability distribution for Sicherman Dice



	1	3	3	5
1	2	4	4	6
2	3	5	5	7
2	3	5	5	7
3	4	6	6	8

P(2)=1/16, P(3)=2/16, P(4)=3/16, P(5)=4/16, P(6)=3/16, P(7)=2/16, P(8)=1/16

An application of Algebra proves this is the only alternative with this property

Representing a dice algebraically as a polynomial where the coefficient multiplying x^n corresponds to the number of ways n can be thrown. For instance, one would represent an ordinary dice as

$x^4+x^3+x^2+x^1$

Note:

- Setting x=1 in the corresponding polynomial will always give you 6.
- You never get a term x^0 in the corresponding polynomial.

Rolling pairs of dice gives the same answer as multiply the polynomials corresponding to the two dice together.

$(x^4+x^3+x^2+x^1)\ (x^4+x^3+x^2+x^1)$

$=x^8+2x^7+3x^6+4x^5+3x^4+2x^3+x^2$

$=x^2(x^4-1)^2/(x-1)^2$

$=x^2(x^2+1)^2(x+1)^2$

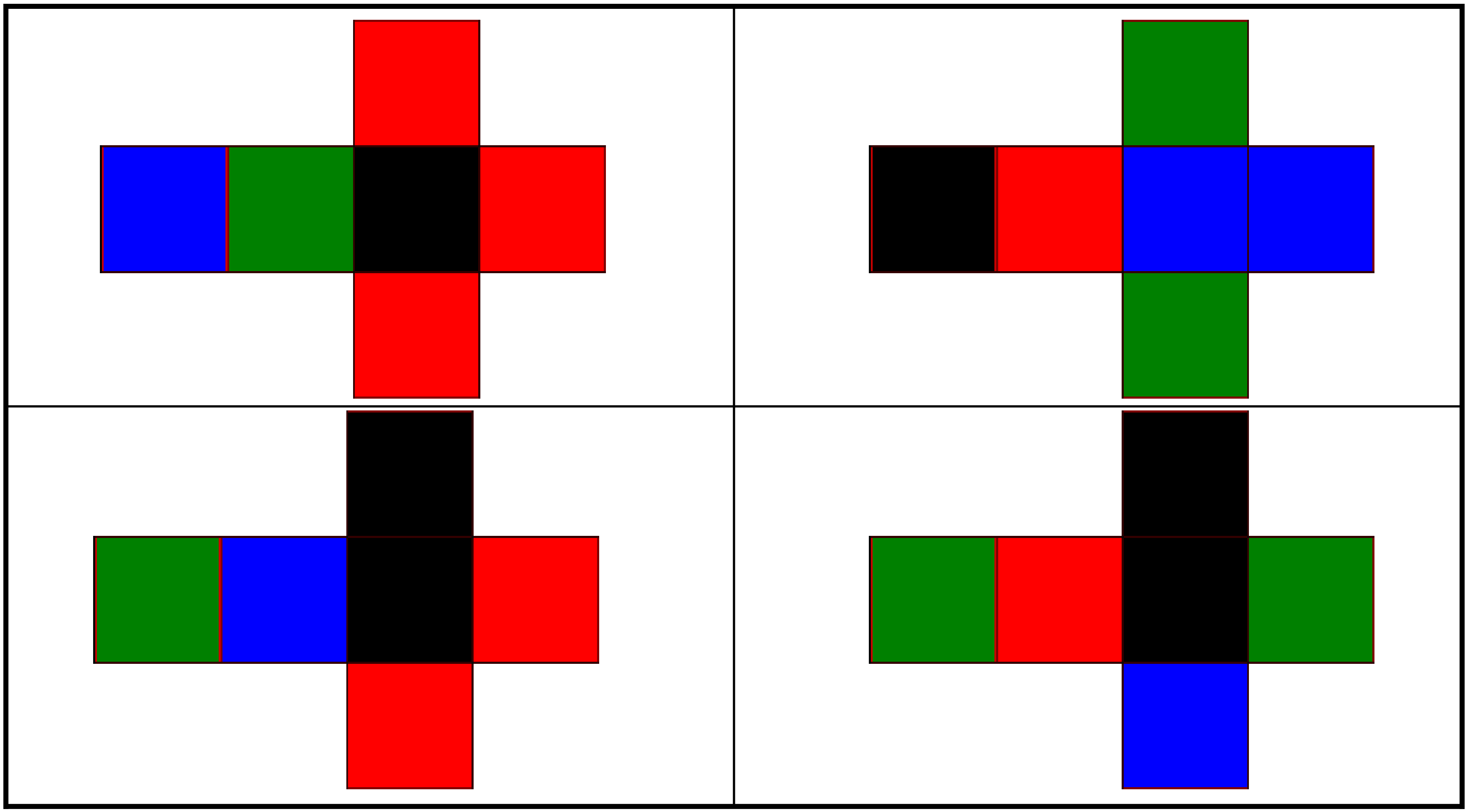
$=[x(x^2+1)^2][x(x+1)^2]$

Uniform distribution

- Can you design a pair of tetrahedral dice whose distribution is uniform:
  $4x^1+4x^2+4x^3+4x^4$
  $=(2+2x^2)(2x+2x^2)$
  Are there others?

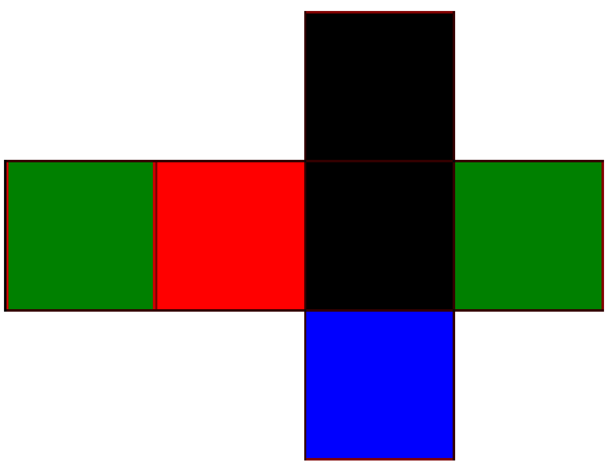
Instant Insanity

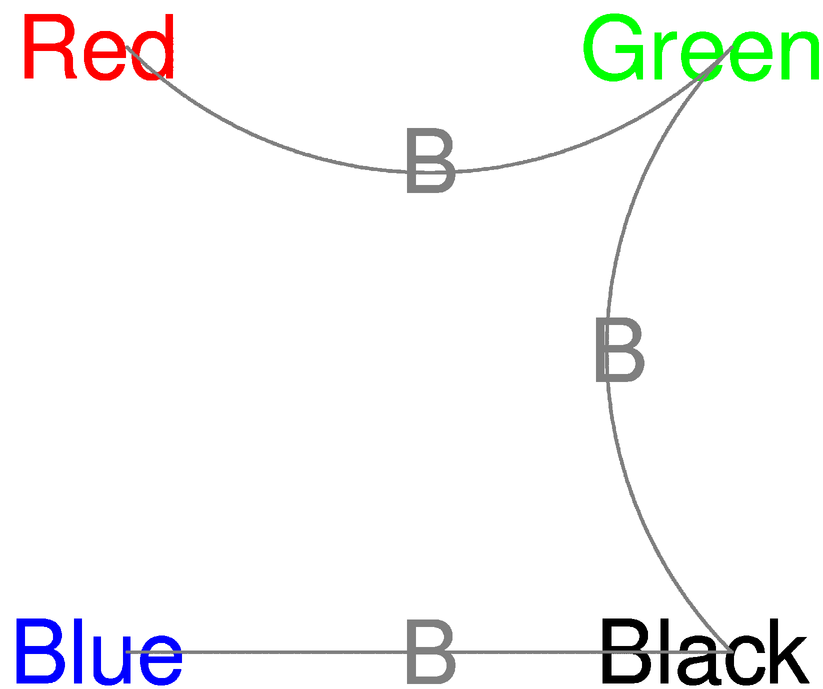
Stack the cubes up so that each of the four colours can be seen on each of the right-left and front back faces of the stack.



Why does it drive you mad!

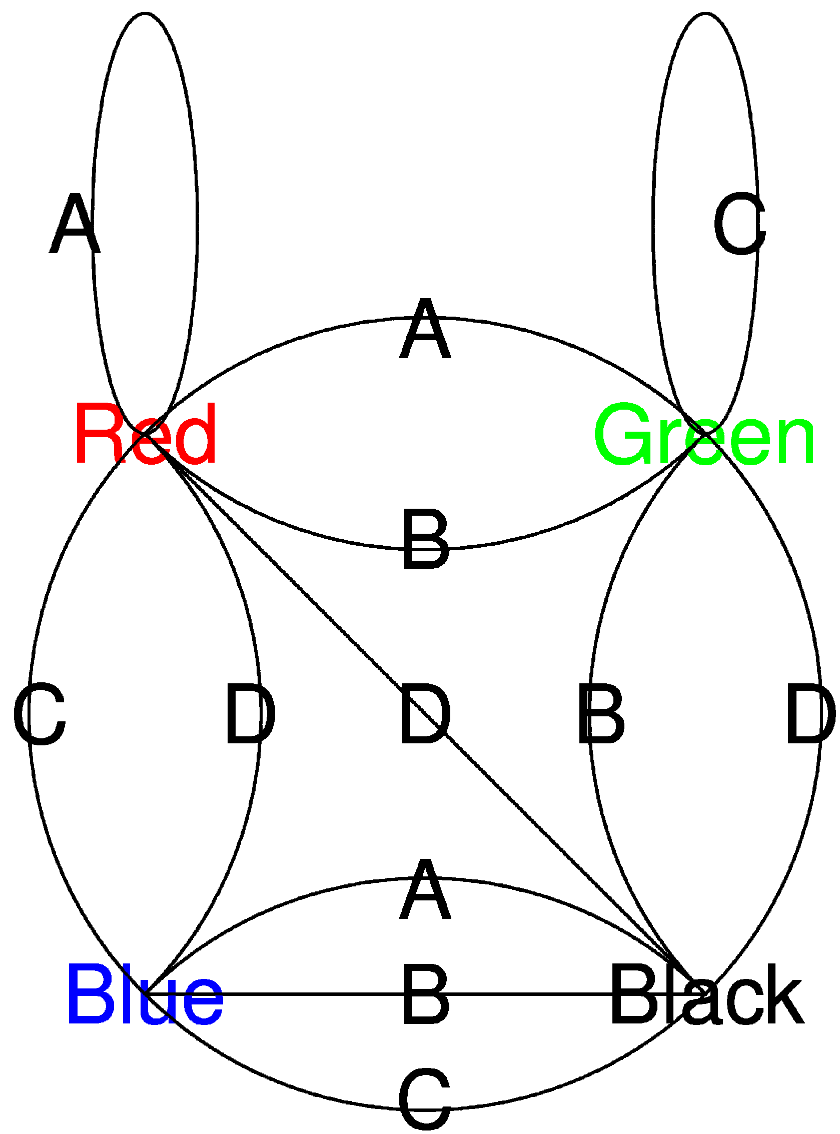
- 24 possible ways of arranging each cube. 24^4 =331776 arrangements.



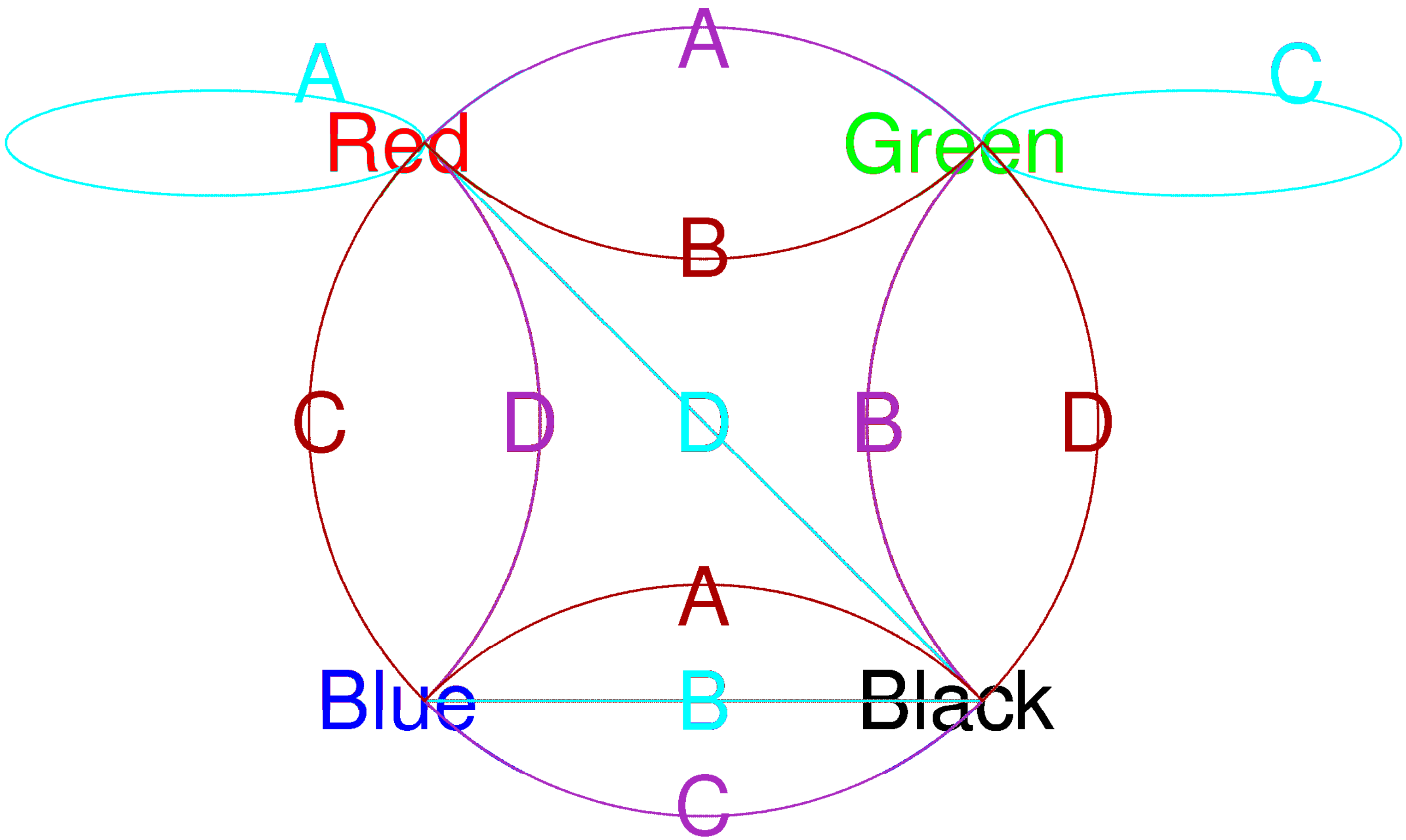


Katzenjammer's puzzle

- Each path visits all of the colours.
- Each letter must appear exactly once along any single path.
- The two paths cannot share an edge.



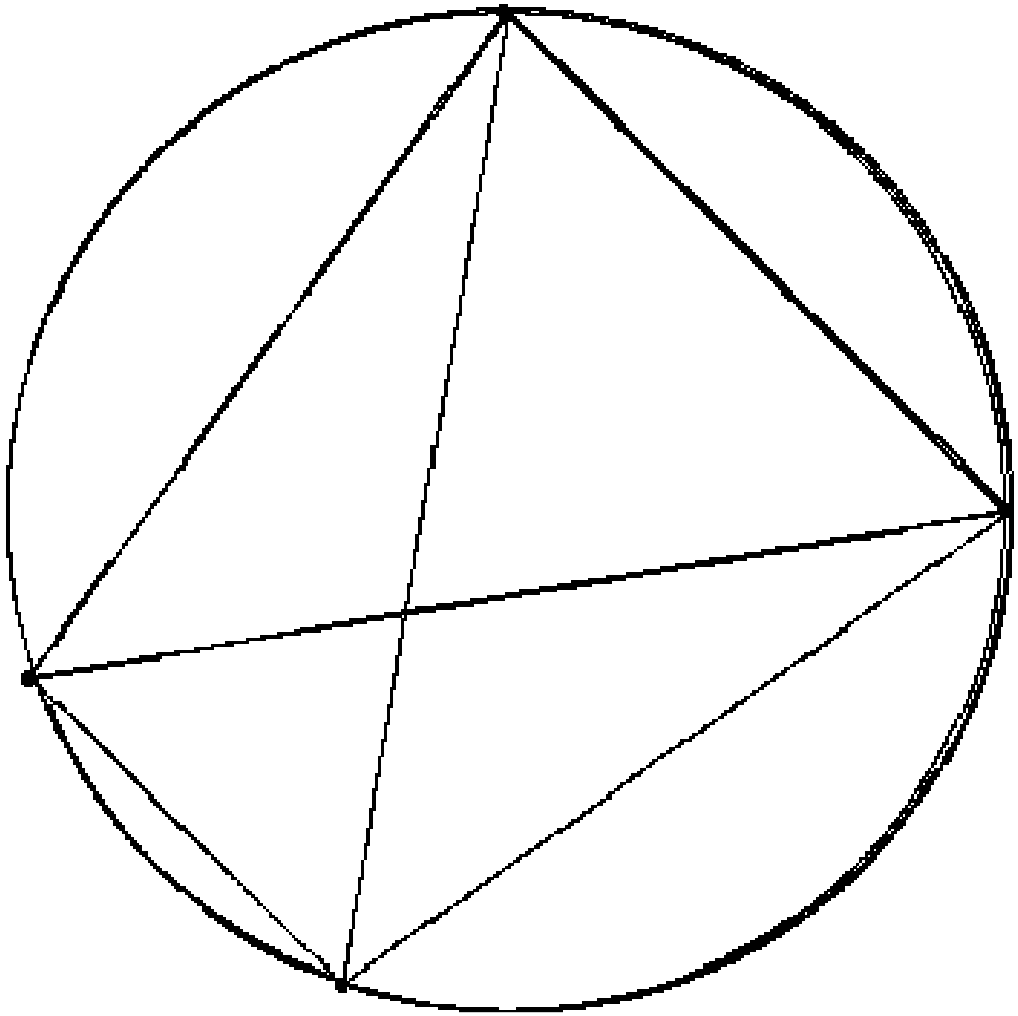
Essentially the only solution is shown below in purple and maroon.



Steiner Networks

- Go to external link

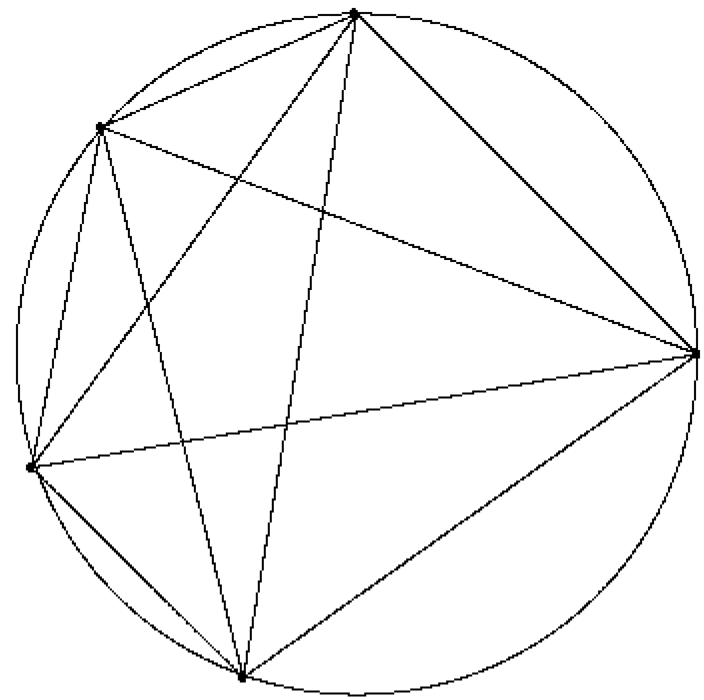
An example of why we need proof: Regions of a circle



#points	#regions
1	1
2	2
3	4
4	8

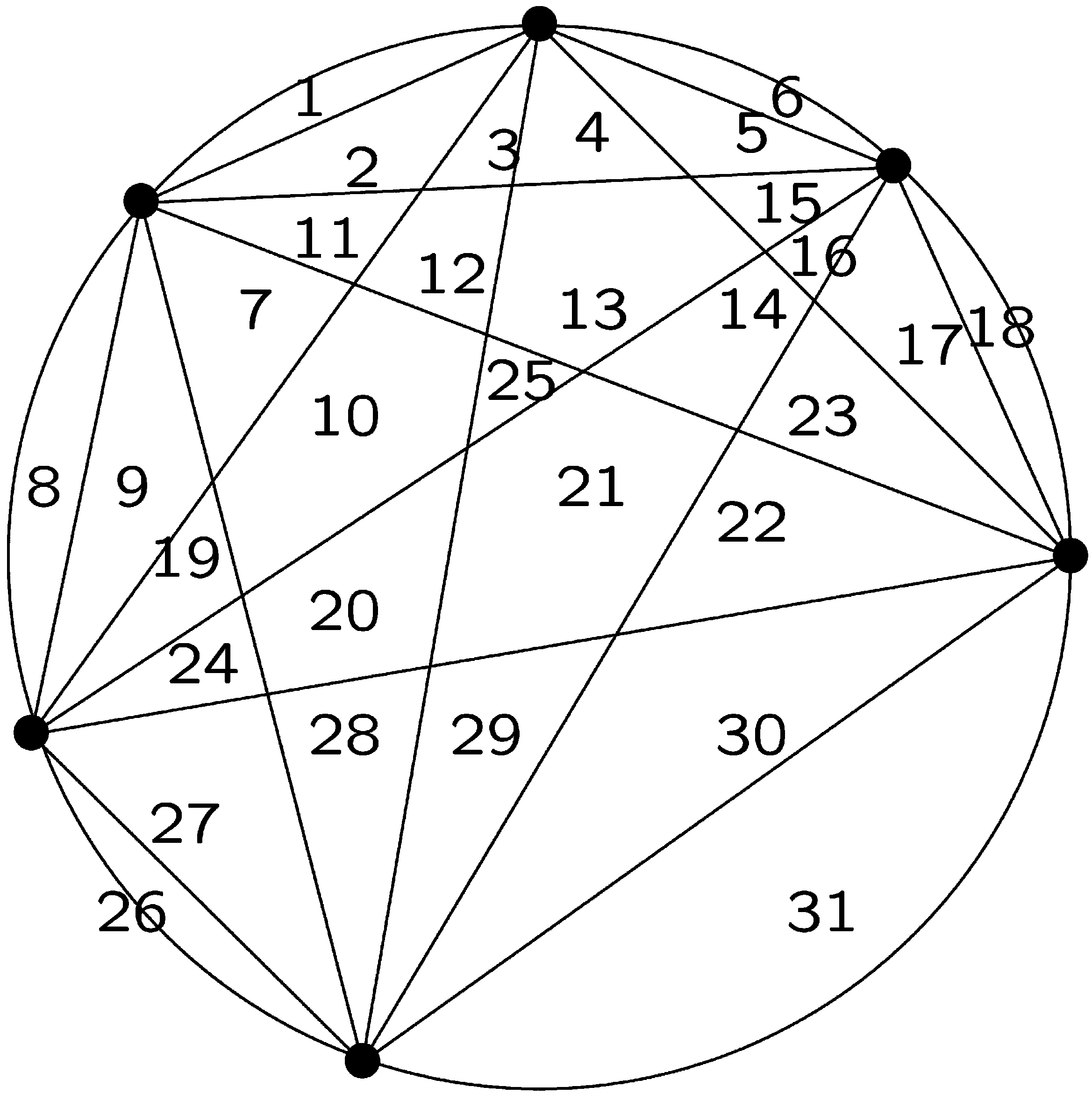
Question 3

With five points on the edge of the circle how many regions do you think there will be?

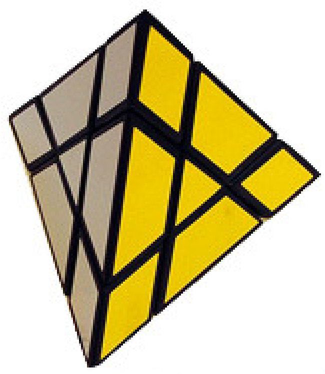


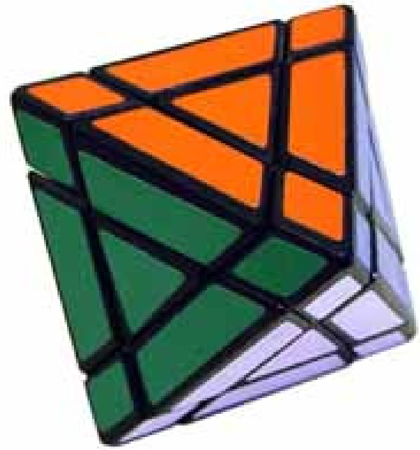
Enter your answer and press Send.

With six points on the edge:



What do the other Rubik Platonic solids look like?





Emmanuel College: Rubik's Puzzles!

– who am I?

The are **519,024,039,293,878,272,000** permutations of the cube and only "12" solutions.

Lots of Maths & engineering.

Is my brain big enough to memorize all possible solutions?

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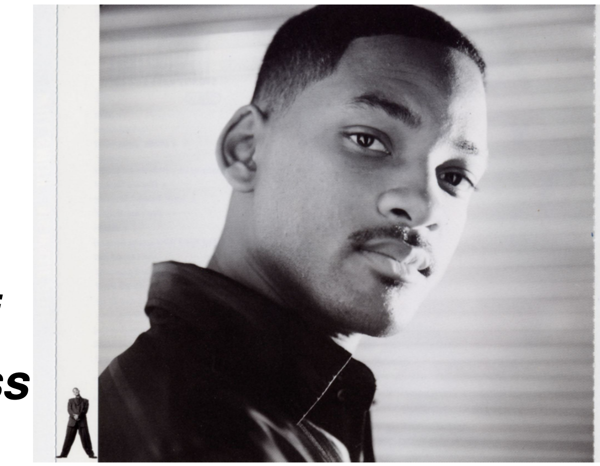
Who is the odd one out?

Michel Gondry (Acclaimed Music Video Director) 	**Will Smith** ***The Pursuit of Happyness*** 
Steve Cram 	**Homer Simpson** 

Using cubes in teaching

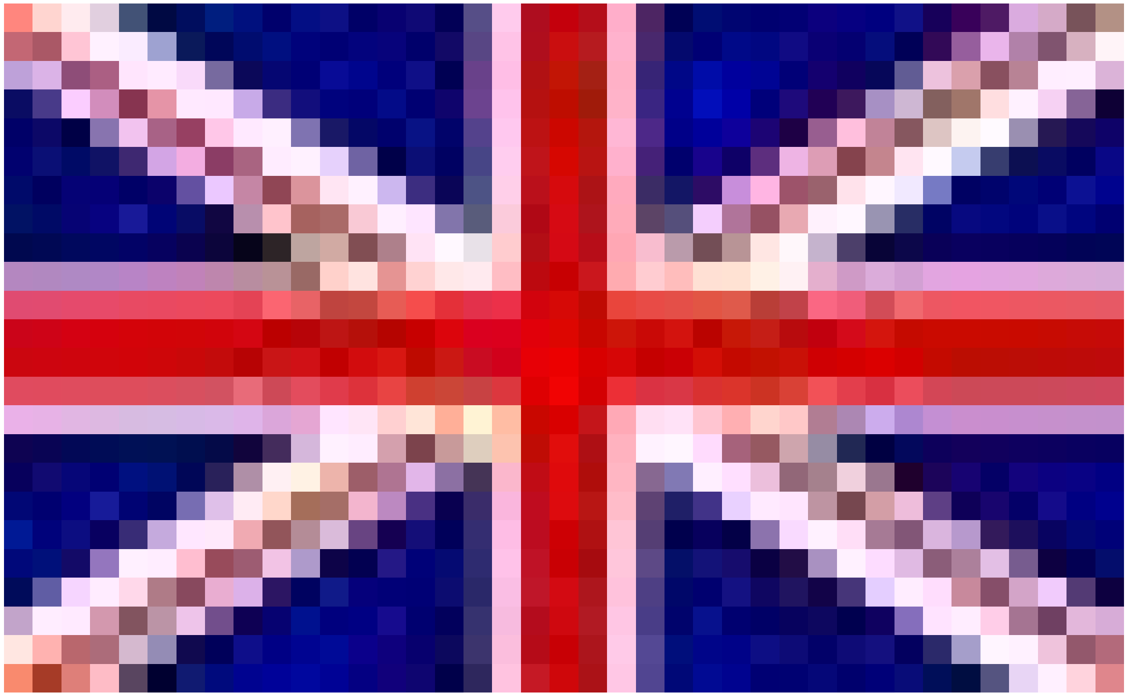
- Discovering “squared” and “cubed”
- $1+3+\ldots+(2n-1)=n^2$
- $1+2+3+\ldots+n=n(n+1)/2$
- Minimum number of moves is currently 22.

Rubik's Cubism of the last supper –
a world record using 4,050 Rubik's style cubes on

a panel measuring (5 m x 2.5 m)

The union flag

- Convert into something with fewer pixel, but approximately the same aspect ratio



- Use a computer programme which averages to a 39x24 picture.
- Not clean. How do I make my own picture?

Automated programme

- I wanted to write a computer programme which could automatically generate numbers and letters to create a picture. For instance a “T” could be represented in 5x3 matrix form as:

1	1	1
0	1	0
0	1	0
0	1	0
0	1	0

Labelling the cubes

- Three examples from the Hilbert Hotel and the Infinite Bus Company Limited.
  - One new customer.
  - One bus of load.
  - A countable number of buses

Mathematics of Cubism

- Colours
- Programming language
- Labelling
- Transferrable Skills

Group Properties

There are six basic permutations of the cube which are clockwise quarter turns of the six sides.

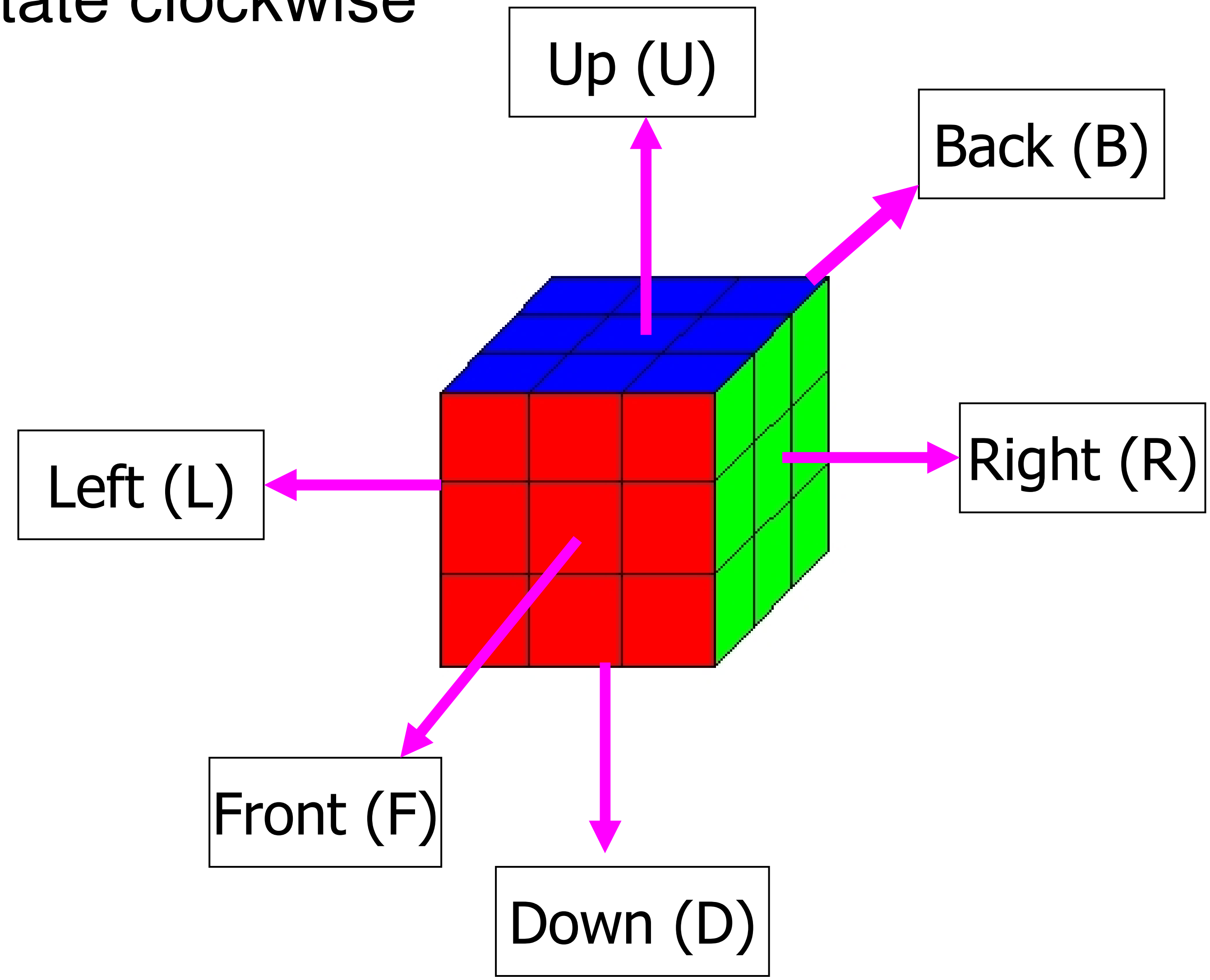
- **Closure**; if we combine two sequences we get another sequence that still belongs to the cube group. For example, **(FRU)** and **(RUL)** are both in the cube group and so is **(FRU)(RUL)**
- **Associative**; if we combine three elements, both ways of combining the elements are the same. For example, **(FB)L** = **F(BL)**
- **Identity**; an element **I** which when combined with any other element gives that element again. For example, **RI=R** and **IR=R**
- **Inverse**; every sequence of moves can be done backwards and therefore undone. For example, $\mathbf{RR^{-1}=I}$ and $\mathbf{R^{-1}R=I}$

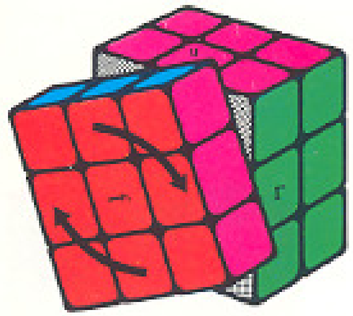
Example: two corner rotation

– Sub-group of order 3 with $(R^{-1}D^{2}RDR^{-1}DR)(L^{-1}D^{2}LDL^{-1}DL)$

Aka Right-Left face corner twist

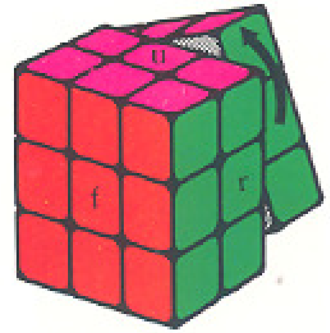
We can adapt ideas. We start by needing an orientation – thankfully centres are fixed. We always rotate clockwise

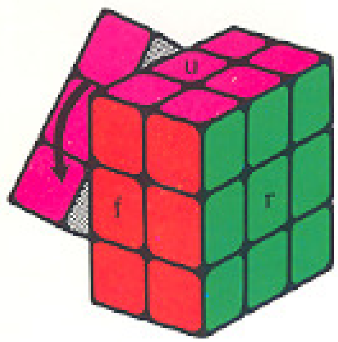




F front

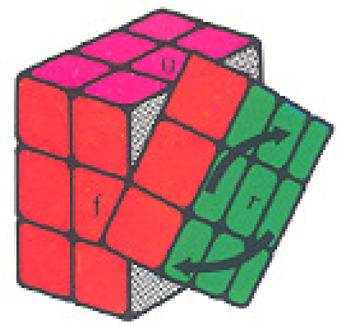
B back

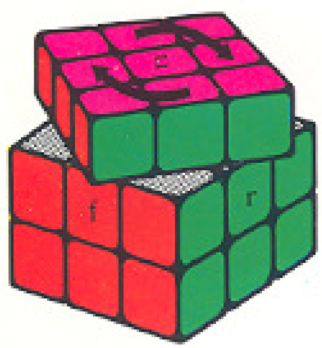




L left

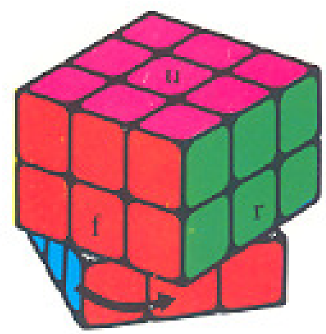
R right





U up

D down



Infinity and Countability

Assume that

$$S = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$

is finite, then

$$\begin{aligned} S &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots \\ &> \underbrace{\frac{1}{2} + \frac{1}{2}}_{=1} + \underbrace{\frac{1}{4} + \frac{1}{4}}_{=\frac{1}{2}} + \underbrace{\frac{1}{6} + \frac{1}{6}}_{=\frac{1}{3}} + \cdots \\ &= S \end{aligned}$$

so that $S > S$ which is a contradiction

You can count the positive rationals

$$
\begin{array}{cccccc}
\frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \cdots \\
\frac{2}{1} & \frac{2}{2} & \frac{2}{3} & \frac{2}{4} & \frac{2}{5} & \cdots \\
\frac{3}{1} & \frac{3}{2} & \frac{3}{3} & \frac{3}{4} & \frac{3}{5} & \cdots \\
\frac{4}{1} & \frac{4}{2} & \frac{4}{3} & \frac{4}{4} & \frac{4}{5} & \cdots \\
\frac{5}{1} & \frac{5}{2} & \frac{5}{3} & \frac{5}{4} & \frac{5}{5} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}
$$

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www.mathscareers.org.uk/

Who are the following & what do they have in common?

- **Science ‘not for normal people’**

  Sometimes you see beautiful people with no brains. Sometimes you have ugly people who are intelligent, like scientists.

- **UK needs lots more maths graduates**

  actuary | **computer game designer** | **statistical consultant** | **systems administrator** | **avalanche researcher** | **medical statistician** | **aerodynamicist** | **meteorologist** | **audio software engineer**

- **Maths graduates *can* earn more**

  On average graduates earn 160K more than those without while Maths graduates earn 225K.

- **Universities have Grant schemes to help cover fees**