Algebraic Topology (Math 4161)

Problem Class 3  16.02.15

This set of problems will be discussed in the Problem Class on 19.02.15, along with homework problems.

1. Let \( 0 \rightarrow A \overset{i}{\rightarrow} B \overset{j}{\rightarrow} C \) be an exact sequence of abelian groups and \( G \) an abelian group.

   (a) Show that there is an exact sequence
   \[
   0 \rightarrow \text{Hom}(G, A) \overset{i^*}{\rightarrow} \text{Hom}(G, B) \overset{j^*}{\rightarrow} \text{Hom}(G, C)
   \]

   (b) Assume that \( j \) is surjective. Show that there is an exact sequence
   \[
   0 \rightarrow \text{Hom}(G, A) \overset{i^*}{\rightarrow} \text{Hom}(G, B) \overset{j^*}{\rightarrow} \text{Hom}(G, C) \rightarrow 0
   \]
   \[
   \quad \text{Ext}(G, A) \overset{i^*}{\rightarrow} \text{Ext}(G, B) \overset{j^*}{\rightarrow} \text{Ext}(G, C) \rightarrow 0
   \]

   (c) Assume that \( j \) is surjective. Show that there is an exact sequence
   \[
   0 \rightarrow \text{Hom}(C, G) \overset{j^*}{\rightarrow} \text{Hom}(B, G) \overset{i^*}{\rightarrow} \text{Hom}(A, G) \rightarrow 0
   \]
   \[
   \quad \text{Ext}(C, G) \overset{j^*}{\rightarrow} \text{Ext}(B, G) \overset{i^*}{\rightarrow} \text{Ext}(A, G) \rightarrow 0
   \]

2. Let \( X \) be a topological space, and \( \varphi, \psi \in C^1(X; \mathbb{K}) \) be cocycles. Show that \( \varphi \cup \psi + \psi \cup \varphi = \delta \chi \) for some \( \chi \in C^1(X; \mathbb{K}) \).