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Communicating Maths III - Magician's Math - 2010/11

General idea

This project is about investigating counterintuitive mathematics results, and reporting on them in your own way, illustrated by your own examples and investigations. Many of these results have to do with combinatorics or statistics, but there certainly are also other mathematical concepts for which humans have bad intuition (e.g. certain aspects of geometry, or the concept of 'infinity', or aspects of graph theory; see the books mentioned below). Some of these lend themselves to use by illusionists, and most are excellent ways to explain dry mathematical theory.

Resist the temptation to simply collect a long list of interesting tricks and facts (you are not writing an encyclopedia). Rather, you should select one subtopic, and then go to some depth into the underlying maths. You can think, for instance, of understanding how the simple examples are relevant for more concrete real-world problems, or how they generalise to larger systems (e.g. arbitrary numbers of cards in a card trick, varying numbers of people in a statistics game, ...) or how they apply when you change the rules of the game (e.g. what happens when you use quantum rules instead of classical ones, or what happens if you use numbers in a different base).

Structure, supervision & meetings

This year the magician's math project has 8 students, split in two groups, one of which is supervised by Kasper Peeters and one by Paul Heslop. At least initially, we will have one or two meetings with everyone together. When you progress and get more into the subtopic of your liking, meetings with fewer students, or one-to-one will be more useful.

One of the main goals of Communicating Mathematics III is that you learn how to find your way through the literature and the maths in it, how to collect interesting material and construct your own examples, and report on all that in your own words. As such, we will not hand you a fixed problem with a canned answer (i.e. these are not lectures).

To get the ball rolling, however, we will organise some meetings a few weeks into the beginning of term, in which each student gives a small presentation (20 minutes or so) about a particular subtopic. The goal of these presentations is to give you some concrete feedback on the directions along which you could extend that presentation into a full report. Depending on which topics you choose, we can then also organise some more lecture-style meetings in we try to get you up to speed with some of the more difficult underlying maths (e.g. a reminder of statistics, or graph theory, or ...).

Literature pointers and summer preparation

If you want to prepare yourself over summer, the first thing to do is to read some of the suggested literature, and pick up a subtopic that you find interesting. If you have doubts about whether it has enough mathematical substance, ask us for advice over email.

If you have more time, start playing with some examples, asking yourself questions like

- Can this be generalised to other cases or related problems?
- Can I reproduce the graphs/data/estimates/... myself, perhaps with some computer help?
- Are there any useful applications (does the 'trick' extend to something with practical value)?
- Do I understand all the underlying maths?
- Is this just an isolated example, or is there some underlying deeper structure which links it to other cases?

Useful starting points are for instance

- Michael Kleber, "The best card trick", http://www.apprendre-en-ligne.net/crypto/magie/card.pdf. This is just one example of a magician's trick which is surprising but mathematically carefully tuned so that it works. There are many versions of the trick, using more cards, different sets of cards, and so on. Google around if you like this.
- Julian Havil has written two books with many examples of non-intuitive maths (not all are equally deep): "Impossible?: Surprising Solutions to Counterintuitive Conundrums" and "Nonplussed!: Mathematical Proof of Implausible Ideas". Several of them have nothing to do with statistics, and many have some real-world application.
- The Derren Brown trick is based on the concept of non-transitive games, and there are many other games which exhibit this, ranging from rock-paper-scissors, which is clearly not terribly deep, to the game of 'go', which has led to a large number of publications, e.g. Cazenave and Helmstetter, "Search for transitive connections" (use Google Scholar to get a pdf copy).
- Martin Gardner has written an almost endless list of articles on surprising mathematics results, some of which are deep enough to be used in this project (there's even an annual conference dedicated to his work).

Your own work

While a CMIII project is not required to contain new results, you *are* expected to think of original applications and examples which illustrate the mathematics which you are studying. You can do that in various ways: create variations on existing examples, do your own data analysis, write small computer codes to do computer experiments, make your own plots of boring data, and so on. Do not just repeat and retell what you learn in books and papers, but think for yourself and constantly ask yourself questions about the material.