Project III 2025-26
Penrose Tilings
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In the early 1970's the mathematician and physicist Roger Penrose developed a series of tilings of the plane with some remarkable properties. In essence his tilings use a finite number of (in many cases just two) shapes of tile which fill the plane in a highly structured manner, but in a way that is completely non-repeating. More specifically, they have no translational symmetries but all finite patches – however large you choose them – repeat themselves infinitely often throughout the resulting tiling. Such objects had been known to exist since the early 1960's (when they arose in the study of logic and theoretical computer science), but the remarkable property of Penrose's examples were that they used such a small number of shapes of tile and had such simple construction rules.

As these objects were further studied it was found they possessed many other remarkable properties. In particular, they can be constructed in (at least) three very different ways. They are also `refractive' in the sense that an atomic array arranged on the vertices of a Penrose tiling produces a sharp `pure point' diffraction image, something that traditionally had been assumed needed a periodic atomic array. This in turn led to their application as models for the quasicrystals discovered ten years later, and subsequently found to occur in nature.

This project aims at exploring these tilings and some of their extraordinary properties. No specific prerequisites are needed other than a familiarity with first and second year level mathematics and an interest in things geometric.

To read a little about Penrose' tilings, the *Wikipedia* page on the topic gives a nice introduction. You can also take a look at
- Branko Grunbaum and G. C. Shephard, *Tilings and Patterns: An Introduction*, W.H.Freeman & Co (1989) (in University Library)
- Marjorie Senechal, *Quasicrystals and Geometry*, CUP (1996)

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