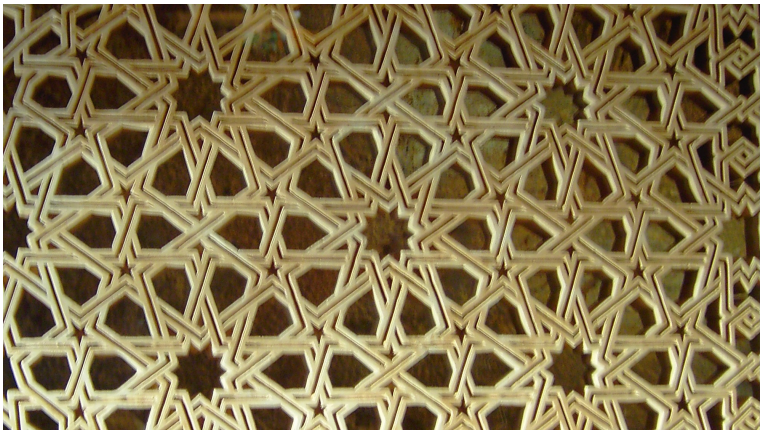
CONWAY'S MAGIC THEOREM: PATTERNS AND WALLPAPER DESIGN

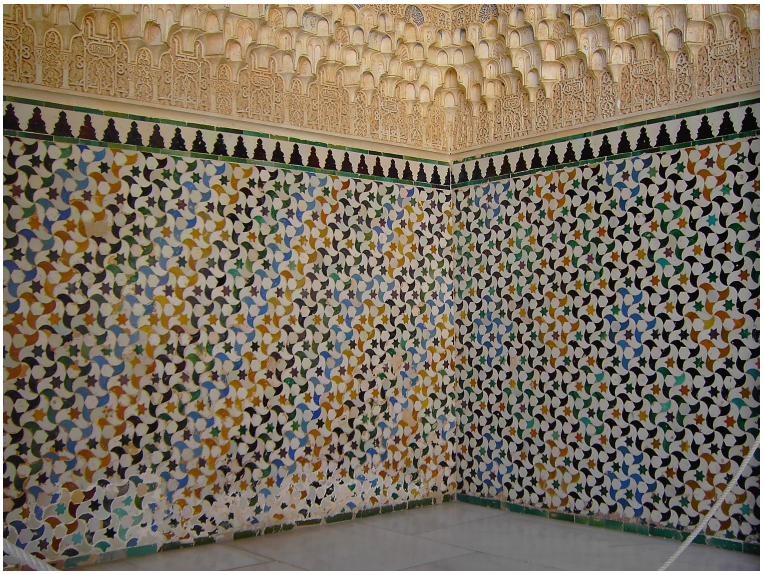
Supervisor: John Hunton
Project research area: Geometry and Topology

Description

Practically speaking, when you design some wallpaper you need a pattern that repeats itself. But there is quite a lot of choice as to how that repetition can work, with the basic motif perhaps just repeating translationally, or with reflections, or rotations, or some mixture of them all. In fact there are precisely 17 distinct ways such patterns can be formed: these are the classical *wallpaper groups* known since the 19^{th} century. This project will explore the topic from the much more recent perspective of John Conway's *Magic Theorem* that uses simple topology to characterise and classify the resulting patterns.

Repeating patterns in the plane concern more than just the design of wallpaper. Beautiful examples (it is said of all 17 cases) can be found in the decorations of the great Alhambra palace in Grenada, three of which are shown below.





Conway's insight was that all these geometric patterns could be analysed by considering a simple topological space derived from the basic motif and the ways the overall pattern repeats. Moreover, his approach can be applied to study patterns on other objects, such as spheres, friezes, the hyperbolic plane, and so on. The group project will aim to study and learn how to use his method to identify patterns, and to understand why it works. The individual projects can go in a variety of ways, from extending the application of his theorem to other domains, to looking at non-repeating patterns that lie outside its immediate approach.

Group Project

By the end of the group project we will have learnt

- The basic symmetries that can arise in a repeating pattern in the plane.
- The statement of Conway's Magic Theorem.
- How to apply his Theorem to identify and create examples.
- Why his Theorem works.

Mode of operation and evidence of learning for the group project

This project will focus mainly on working through the first six chapters of Conway, Burgiel and Goodman-Strauss' recent book *The Magic Theorem* (available digitally from the library.) Students will demonstrate their understanding by solving the exercises it offers, identifying the symmetries in given patterns and exploring the underlying theory that makes it work. As with other topics, this will include clearly communicating these skills and ideas in both written and oral formats.

Individual Projects

The individual projects will extend or complement the ideas of the group project. For example, possible routes to go could be

- Further extending applications of the Magic Theorem to other spaces (as detailed in the core book)

- Exploring further the topology used to prove the Magic Theorem.
- Looking at links with Group Theory and more algebraic approaches to the classification of wallpaper groups.
- Considering analogues of the result in higher dimensions (for example the so-called *crystallographic groups* in dimension 3).
- Studying structured but non-repeating patterns in the plane, such as the Penrose tiling, that do not fit the Magic Theorem.

Mode of operation and evidence of learning for the individual project

The individual projects will typically focus on reading material from a variety of texts relevant to the topic chosen. The emphasis will be on mathematical precision and the development of deep conceptual understanding. Students will demonstrate their understanding though exploring examples and communicating clearly the theory and examples in both written and oral formats.

Pre-requisites and co-requisites

The key pre-requisite is perhaps a general interest and comfortableness in thinking about geometric patterns and ideas. As the focus will include mathematical rigour and precision, the style of modules such as Algebra II and Complex Analysis II are most relevant, though only limited use will be made of explicit results from them.

There are no co-requisites but students, depending on which individual topic they pursue, may find some interesting parallels with parts of the modules Galois Theory, Groups & Geometry III, Codes & Knots III or Analysis & Topology III.

Additional information

If you would like more information about this project, discuss its scope and/or its pre- and co-requisites, please contact me at john.hunton@durham.ac.uk.

Resources

The key text for the Group project will be the book Conway, Burgiel and Goodman-Strauss, *The Magic Theorem: a greatly-expanded, much-abridged edition of 'The symmetries of things'*, available digitally from the library.

As the subtitle suggests, this is an abridged version of an earlier book *The Symmetries of Things* by the same authors, also available digitally from the library, and which may be of some background interest, including for a number of the individual project topics.

In general for the individual project, the relevant resources will depend on the direction someone wishes to go in, but a good starting point for many routes (beyond the texts above) would be the classic book Grünbaum & Shephard, *Tilings and Patterns*.