PROJECT IV 2024-5

COMPLEX PATTERNS VIA TOPOLOGY

Mathematics is very good at describing both phenomena that are very symmetric, and phenomena that are pretty random, but there are a lot of things in between: more explicitly, while group theory is good at categorising symmetric patterns, and probability and statistics for more random or indeterminate systems, the idea of a pattern that is close to being symmetric, but not quite, is better thought about using *topology*. This project will use powerful ideas from topology to investigate a variety of interesting objects that are in this intermediary area, neither completely symmetric nor completely random.

Examples of the sort of patterns we have in mind might be patterns in the plane such as the Penrose tiling that have rich local structures, but no global symmetries, or infinite sequences of numbers or letters whose finite subsequences keep repeating, but without any global repetition: such objects arise in geometry, number theory and theoretical computing. Yet more examples arise in dynamics, thought of as states of a system that evolve over time: for example, the long-term behaviour of a seemingly `chaotic' system, or for another example, the position of all the planets in the sky today may be a pattern that will never exactly come again (so, does not repeat exactly over time), but something close to today's configuration may occur on many future (and past) occasions.

This project will look at examples of such complex patterns, how their properties can be translated into topological terms, and how tools from topology help to examine them.

ESSENTIAL PRIOR OR COMPANION MODULES

Topology III - MATH3281 would be useful. The module Algebraic Topology IV – MATH4161, could be helpful if it was also taken. If interested, the module Ergodic Theory IV – MATH4361 would suggest other ways in which this project could be taken.

RESOURCES

N. Frank, *A primer of substitution tilings of the Euclidean plane*, Expo. Math. 26 (2008), no. 4, 295-326. Also available via her webpage https://pages.vassar.edu/nafrank/?page_id=15

M. Senechal, *Quasicrystals and Geometry*, Cambridge University Press (1996);

L. Sadun, *Topology of Tiling Spaces*, American Mathematical Society, University Lecture Series 46 (2008).

The Wikipedia article on Aperiodic Tilings https://en.wikipedia.org/wiki/Aperiodic_tiling also gives an excellent overview of the sorts of objects we can use topology to analyse.

There are the slides of a nice general talk introducing the subject at http://www.math.uvic.ca/faculty/putnam/r/UNR_colloquium.pdf

For some background on complex patterns, try the classic

- Branko Grunbaum and G. C. Shephard, *Tilings and Patterns: An Introduction*, W.H.Freeman & Co (1989)

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