PROJECT III 2024-25

Cutting Sequences and Mathematical Billiards

Here is a simple example. Imagine the plane filled with a square grid and a straight line L is drawn across it in such a way that it does not pass through any of the corners of the squares. Now record the sequence (the *cutting sequence*) of intersections of this line with the grid, recording if the intersection is where L cuts a vertical line (v) or a horizontal line (h). If the line is at a rational slope with respect to the grid then this will be a periodic sequence of v`s and $h$`s. But if it is not, and L lies at an *irrational* angle, then the resulting sequence is much more interesting and has a large number of rather curious properties.

From another perspective, if you hit a billiard ball on a square (perfectly cushioned) billiard table and followed its path as it bounced around, the pattern would be modelled by the sequence above: the link being that square grid is in a sense an `unfolding' of the table, a way of modelling the bounces as equivalent to passing through to a new table on the other side of the cushion.

The resulting mathematics analyzing sequences such as these (and related constructions) has powerful links with combinatorics, surfaces and their foliations, number theory, continued fractions, dynamics, topology and mathematical billiards. This project can be taken in a number of ways and will explore both examples and theory related to these ideas.

ESSENTIAL PRIOR OR COMPANION MODULES
Basic background in Level 2 pure mathematics modules would give sufficient prerequisites to study this project.

RESOURCES
There are many good resources on the web. Searching for basic articles *mathematical billiards* on Wolfram Mathworld, or *Sturmian words* on Wikipedia would certainly give some good introductions.

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