Level 4 Project

The Fundamental Group and `Interesting' Spaces

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Undergraduate topology modules usually study only relatively simple spaces such as manifolds or simplicial complexes, but most topological spaces are more complex. For example, spaces that arise in chaotic dynamics are usually highly structured, but have none of those easy descriptions; in fact they can often be considered as *limits* of such basic objects in some very precise sense. For a classic example, have a look on the internet at pictures of the *Lorenz Attractor* – in fact the background of this page is one such picture (with thanks to Wikipedia).

This project looks at developing and applying algebraic topological methods – such as the fundamental group, though others are possible too – to these types of limit spaces. There are a variety of ways it could go, but by and large the work will draw on a mix of topology, and algebra and maybe some geometry if you wish.

A (very) short sketch of some of the basic issues can be found on the Wikipedia page on Shape Theory.
Discussion of limits in general and the application of the fundamental group can be found in Ross Geoghegan's book *Topological Methods in Group Theory*, CUP, Springer 2008 – see sections 11.2, 11.3 for limits, Chapter 16 for the fundamental group and Chapters 11 and 12 for other algebraic topological machinery. (Incidentally, the earlier parts of this book may also be of interest if you are taking Algebraic Topology IV.) It can be found in the library, at 512.55 GEO.
A discussion of the construction of the spaces we're interested in, their link to chaotic dynamics and the use of the fundamental group can be found in A.Clark & J.Hunton *Tiling spaces, codimension one attractors and shape* New York Journal of Mathematics, 18 (2012) 765-796, available here.
An elementary discussion of one way of approaching a particular class of such spaces in the spirit of this project is in Lorenzo Sadun's book *Topology of Tiling Spaces*, American Maths Society, University Lecture Series 46 (2008).

Prerequisites: Topology III is probably essential, but other than that a background in pure maths is all that is needed. It might be interesting (though certainly not necessary) to take this project if studying also one or both of Algebraic Topology IV or Ergodic Theory IV either of which would complement it well.

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