FUSE: A rainfall runoff model

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April 30, 2014

1 Model Outline

The FUSE model[1] uses rainfall and evapotranspiration input to produce a time series of estimated streamflow discharge. The model can be represented as a function

$$Q(x, s, P, E)$$

where s is a vector of 8 model structure choices $s_1, \ldots, s_8$ that determines how the water storage is modelled and what equations to use to calculate the various fluxes in the system; x is a vector of 22 inputs $x_1, \ldots, x_{22}$ that give the values of the various unknown parameters in the equations specified by s; P is a time series of average rainfall over the catchment; and E is a time series of potential evapotranspiration. The output is a time series of discharges, with time step equal to that in P and E (hence, these two forcing functions must be synchronized correctly).

1.1 Model Structure

The model structure choice s is a collection of switches that control the details of the model to use. First it should be noted that we are not using s_1, which controls how rainfall error is handled (we intend to explicitly handle rainfall error by other means), but we leave it in the notation for completeness and also to match up with the notation in the `R` wrapper.

The remaining elements of s deal with two main aspects of the model: the structure of the model of the water storage (s_2 and s_3), and the equations for the various flows between the compartments of the storage model (s_3–s_7).

We first consider the water storage model. The FUSE model splits the storage into two layers: the upper layer and the lower layer. Each layer is then split into several zones.

The structure of the upper layer is governed by the switch s_2. Three possible structures are available, hence s_2 can take three values. The simplest structure is just a single zone. Another option is to have two zones: a tension zone and

[1]Since FUSE contains many different model *structures*, it could be considered a model ensemble rather than a model. For the purpose of this document, however, we shall refer to it simply as a model.

a free zone. In such a system, rainfall enters the tension zone, from which it only leaves through evaporation (such water is lost from the system) or overflow (when the tension zone has reached its capacity). This overflow moves to the free zone, from where it is free to move to the lower layer through percolation. The third possibility is two have two tension zones and a free zone. In this structure, the tensions zones are in series: overflow from the first tension zone goes into the second tension zone, and overflow from the second tension zone goes into the free zone.

The structure of the lower layer is governed by the switch s_3. Again, there are three structures possible. In the first, the lower layer is a single zone of unlimited size. In the second, the lower layer is a single zone but with a finite size. In this third, there is a tension zone that operates in the same manner of those in the upper layer, and two free zones that are in parallel: water overflowing from the tension zone is split between the two free zones. Note that although there are three choices for lower-layer structure, s_3 has four possible values, because it governs not just the structure of the lower layer but the behaviour of the *base flow*—if the lower layer is a single zone of unlimited size, the baseflow can be estimated in two ways: as a fraction of the water content in the lower layer (constant) or characterized by a power recession function.

We now move on to the switches controlling the equations of flow (s_3 to s_7).

- *Precipitation* is the only source of water in the model. It is not governed by any switches and parameters—the amount of water entering the system at a given time is simply the value of the time series P at that time. Note that, were we to be using the rainfall error switch s_1, the actual precipitation would be a portion of the measured precipitation.

- *Evapotranspiration* is a flow of water out of the system. We write e_1 for the upper layer and e_2 for the lower layer. When the upper layer has two tension components, we write e_1^A and e_1^B. It is governed by switch s_6.

- *Percolation* is the flow of water between the upper layer and the lower layer. We write it as q_{12}. It is governed by switch s_5.

- *Interflow* is a lateral flow of water from the upper layer, which eventually contributes to runoff but slowly and with relatively small influence. We write it as q_{if}. It is governed by s_7.

- *Surface runoff* is a proportion of the rainfall that does not reach any of the zones in the model, instead being immediately released as discharge. This is because Clark et al. (2008) define the runoff as a saturation-excess mechanism. We write it q_{sx}. It is governed by s_4.

- *Overflow* occurs when a zone has reached its capacity. Some water leaves the saturated zone and enters the next zone in the layer. If the final zone of the upper layer overflows, then the overflowing water becomes runoff. If the final zone of the lower layer overflows, then the overflowing water becomes baseflow. We write this flow using a superscript to denote the layer and a subscript to denote the zone that the overflow is coming *from*. For example, in the structure with a single upper-layer tension zone, overflow from this zone is written q_1^T; overflow from the second free zone in the lower layer is written q_2^{FB}.

- *Base flow* is a flow of water from free zones in the lower layer to discharge. We write it q_b; if there are two free zones in the lower layer we write q_b^A and q_b^B.

Finally, s_8 controls neither model structure nor flow but rather whether runoff reaches the river instantly or there is a time delay.

We can now give a summary of the flows affecting each zone.

- In the first upper-layer tension zone, the flow in is P and the flows out are q_{sx}, e_1 (or e_1^A), and q_1^T (or q_1^{TA}).
- In the second upper-layer tension zone, the flow in is q_1^T and the flows out are e_1^B and q_1^{TB}.
- In upper-layer free zones, the flow in is q_1^T (or q_1^{TB}), and the flows out are q_{if}, q_{12}, and q_1^F.
- In upper-layer single zones, the flow in is P and the flows out are q_{sx}, e_1, q_{if}, q_{12}, and q_1^F.
- In lower-layer tension zones, the flow in is a proportion of q_{12} and the flows out are e_2 and q_2^T
- In lower-layer free zones, the flows in are a proportion of q_{12} and half of q_2^T (half because there are two free zones), and the flows out are q_b^A and q_2^{FA} (or q_b^B and q_2^{FB}).
- In lower-layer single zones, the situation depends on whether the size of the zone is unlimited. If it is, then the flow in is q_{12} and the flow out is q_b. If it is limited, then the flow in is q_{12} and the flows out are e_2, q_b, and q_2^F.

1.2 Input Parameters

21 of the 22 input parameters appear in the equations to calculate the flows described above. The final input parameter is used in the calculation of the time delay. For a particular choice s, only a subset of the parameters will have influence, because most of the parameters appear in equations that are only relevant for certain values of the switches. Only parameters x_1, x_2, and x_3, are required for all possible s. The smallest number of parameters active is 7 and the largest is 15. Table 1 shows the parameters, their `RHydro` names, their notation from the Clark paper, a description of their role, and what model structures they are used in.

1.3 Forcing Functions

The rainfall forcing function should be the average rate of rainfall over the whole catchment. In practice, rainfall data will be gathered at particular points using rain gauges, and these observations will be interpolated according to some model to give an average for the catchment. The models to use for this process will be examined in another document. Another possibile way of constructing the rainfall forcing function is to use output from a climate model, but these tend not to be very reliable. Potential evapotranspiration is difficult to measure directly and instead is usually estimated using the Penman-Monteith equation. This equation depends on temperature, humidity, radiation, and wind speed. The discharge output will be on the same time scale as the forcing function time series. Obviously, the two forcing functions must themselves be on the same scale and correspond to the same time periods.

1.4 Output

The output of the model is a time series of discharge. We have available estimates of discharge at several sites in the catchment.

2 Model Structures

Let us now move on to a more detailed examination of the specific effects of switch choices (and in so doing, also see the influences of the various parameters). Each particular combination of switch settings creates a new model (although some combinations do not make sense and are not admitted). In what follows we shall consider each switch, its meaning for the model, and some preliminary explorations of its effects. It should be noted that these explorations are only ad-hoc and should not be given very much weight. In particular, different choices of s lead to different active subsets of x that are active. It can therefore be difficult to compare two different s without trying very many different possible x.

Each switch choice has a particular name in `RHydro`. We examine each in turn and indicate how it matches up with our s_i notation. Before doing so it is useful to fix notation for the various states in the model. Let S_1 be the water content in the upper layer, S_1^T be the tension water content in the upper layer, S_1^{TA} be the tension water content in the primary tension zone of the upper layer, S_1^{TB} be the tension water content in the secondary tension zone of the upper layer, S_1^F be the free water content in the upper layer, S_2 be the total water content in the lower layer, S_2^T be the tension water content in the lower layer, S_2^{FA} be the free water content in the first free tension zone of the lower layer, and S_2^{FB} be the free water content in the second free zone of the lower layer.

2.1 Upper-layer architecture: s_2

The FUSE model splits the ground into two sections: the upper layer and the lower layer. The upper-layer architecture switch determines how the upper layer is composed. There are three possibilities. For $s_2 = 1$ (`onestate_1`), there is only one compartment. The differential equation for the upper layer state variable (that is, the water content of the upper-layer zone) is then

$$\frac{\mathrm{d}S_1}{\mathrm{d}t} = P - q_{sx} - e_1 - q_{if} - q_{12} - q_1^F.$$

For $s_2 = 2$ (`tension1_1`), the upper layer is split into a tension zone and a free zone. The differential equations for the state variables are then

$$\begin{aligned}\frac{\mathrm{d}S_1^T}{\mathrm{d}t} &= P - q_{sx} - e_1 - q_1^T \\ \frac{\mathrm{d}S_1^F}{\mathrm{d}t} &= q_1^T - q_{if} - q_{12} - q_1^F\end{aligned}$$

Finally, for $s_2 = 3$ (`tension2_1`) there are two tension zones in series and one free zone. The differential equations for the state variables are then

$$\begin{aligned}\frac{\mathrm{d}S_1^{TA}}{\mathrm{d}t} &= P - q_{sx} - e_1^A - q_1^{TA} \\ \frac{\mathrm{d}S_1^{TB}}{\mathrm{d}t} &= q_1^{TA} - e_1^B - q_1^{TB} \\ \frac{\mathrm{d}S_1^F}{\mathrm{d}t} &= q_1^{TB} - q_{if} - q_{12} - q_1^F\end{aligned}$$

As one would expect from the equations, $s_2 = 3$ gives results approaching those for $s_2 = 2$ as $x_4 \to 1$. However, the same effect is not observed for $x_4 \to 0$. When comparing the two structures, $s_2 = 3$ seems to respond less to short periods of rainfall. During periods of heavy rainfall or long periods of steady rainfall, the two structures behave approximately the same. The differences within $s_2 = 3$ for different values of x_1, x_3 and x_4 do not seem to follow any readily-identifiable pattern.

The differences between $s_2 = 1$ and $s_2 = 3$ are somewhat complicated. Suppose that x_1 is small, so there is not much storage available in the upper layer. When $x_3 = 0.5$, the two give similar results. When $x_3 > 0.5$, more discharge is observed from $s_2 = 1$. When $x_3 < 0.5$, more discharge is observed from $s_2 = 3$. If instead x_1 is large, it seems that $s_2 = 1$ gives higher discharge in all cases. This difference in discharge tends to be relatively higher during short rainfall events.

2.2 Lower-layer architecture and baseflow

There are similar choices for the structure of the lower layer. For $s_3 = 1$ (`fixedsiz_2`), the lower layer is treated as a single zone. The differential equation for the state variable for this zone is

$$\frac{\mathrm{d}S_2}{\mathrm{d}t} = q_{12} - e_2 - q_b - q_2^F.$$

For $s_3 = 2$ (`tens2pll_2`), the lower layer comprises a tension zone and two parallel free zones. The differential equations for these state variables are

$$\begin{aligned}
\frac{\mathrm{d}S_2^T}{\mathrm{d}t} &= x_{11}q_{12} - e_2 - q_2^T \\
\frac{\mathrm{d}S_2^{FA}}{\mathrm{d}t} &= \frac{1-x_{11}}{2}q_{12} + \frac{q_2^T}{2} - q_b^A - q_2^{FA} \\
\frac{\mathrm{d}S_2^{FB}}{\mathrm{d}t} &= \frac{1-x_{11}}{2}q_{12} + \frac{q_2^T}{2} - q_b^B - q_2^{FB}.
\end{aligned}$$

For both $s_3 = 3$ (`unlimfrc_2`) and $s_3 = 4$ (`unlimpow_2`) the structure of the lower layer is the same: a single zone, but this time of unlimited size. The differential equation for this state variable is

$$\frac{\mathrm{d}S_2}{\mathrm{d}t} = q_{12} - q_b.$$

Observe that there is no evapotranspiration in this equation. The reason for this will become clear when we consider the equations for evapotranspiration.

The structure of the lower layer also determines the equations for the base flow (q_b, q_b^A, q_b^B). For $s_3 = 1$ we have

$$q_b = x_{13}\left(\frac{S_2}{x_2}\right)^{x_{14}}.$$

Note that x_2 is the maximum storage in S_2, so the fraction above is the proportion of the lower layer that contains water. For $s_3 = 2$ we have

$$q_b = x_{16}S_2^{FA} + x_{17}S_2^{FB}.$$

For $s_3 = 3$ we have

$$q_b = x_{15}S_2.$$

For $s_3 = 4$ we have

$$q_b = \frac{x_{13}x_2}{x_{14}}\left(\frac{S_2}{x_2\lambda(x_{14}, x_{20}, x_{21})}\right)^{x_{14}}$$

where

$$\lambda(x_{14}, x_{20}, x_{21}) = \int_0^\infty \frac{e^{z/x_{14}}}{x_{21}\Gamma((x_{20}-3)/x_{21})}\left(\frac{z-3}{x_{21}}\right)^{\frac{x_{20}-3}{x_{21}}-1} e^{-(z-3)/x_{21}}\mathrm{d}z.$$

All four structures seem to give essentially identical results during periods of low rainfall. During heavy rainfall, $s_3 = 3$ gives the highest peaks. This difference is higher when x_{15} is lower. The other three structures have similar peak heights. With $s_3 = 2$ or $s_3 = 4$, the falling limb is very steep, that is, very quickly after a rainfall event the discharge will drop to its “no rainfall” level. The steepness of the limb tends to be very similar for these two structures. Parameters x_{20} and x_{21} do not seem to have much effect on $s_3 = 4$ In contrast, the other two model structures lead to much shallower falling limbs, with discharge continuing for a long time after the rain has stopped. The behaviour of the falling limb in these structures is influenced by parameters x_{13} and x_{14}, but no discernible pattern has been observed. Finally, the “no rainfall” discharge level in $s_3 = 2$ is noticeably higher than the other three structures.

2.3 Surface runoff

For all model structures, surface runoff is directly proportional to precipitation:

$$q_{sx} = AP$$

where A is the *saturated area*. The differences come in the way that saturated area is calculated. It always depends on the proportion of certain zones that are filled with water, but the differences are which zones that is, and exactly what the dependency is. For $s_4 = 1$ (`arno_x_vic`), the relevant zones are all those in the upper layer:

$$A = 1 - \left(1 - \frac{S_1}{x_1}\right)^{x_{19}}.$$

For $s_4 = 2$ (`prms_varnt`), the relevant zones are the upper tension zones:

$$A = x_{18}\frac{S_1^T}{x_3 x_1}$$

where $x_3 x_1$ is the tension storage in the upper layer. For $s_4 = 3$ (`tmdl_param`), the relevant zones are the all those in the lower layer. This choice is much more complicated:

$$A = \int_c^\infty \frac{1}{x_{21}\Gamma((x_{20}-3)/x_{21})}\left(\frac{z-3}{x_{21}}\right)^{\frac{x_{20}-3}{x_{21}}-1} e^{-(z-3)/x_{21}}\mathrm{d}z$$

with

$$c = \log\left(\left(\frac{\lambda(x_{14}, x_{20}, x_{21})x_2}{S_2}\right)^{x_{14}}\right).$$

Initial experiments suggest that $s_4 = 1$ and $s_4 = 2$ are not particularly different: for each choice of parameters for one structure that we tried, we could find a choice of parameters for the other structure that gave very similar results. It also seems that choice of parameters for $s_4 = 3$ can give all manner of behaviour. In particular, overall height of peaks can be controlled by x_{20} and x_{21}. It is not so easy to find choices of parameters that can match output from the other two structures, though, so it seems that $s_4 = 3$ is fundamentally different from the other two in behaviour.

2.4 Percolation

For $s_5 = 1$ (`perc_f2sat`), the percolation is given by

$$q_{12} = x_7 \left(\frac{S_1^F}{(1 - x_3)x_1} \right)^{x_8}.$$

Note that $(1 - x_3)x_1$ is the maximum free storage in the upper layer, so the percolation depends on the proportion of the free storage that is filled. Note also that this switch choice can be made even when the upper layer is a single zone—the model calculates the percolation as if the upper layer were actually split into tension and free (this is why x_3 is active even when the upper layer is only modelled as one zone).

With $s_5 = 2$ (`perc_w2sat`) the equation is very similar but here the total upper-layer is relevant rather than just the free zone:

$$q_{12} = x_7 \left(\frac{S_1}{x_1} \right)^{x_8}.$$

Finally, with $s_5 = 3$ (`perc_lower`) the percolation depends not only on the proportion of water in the free upper layer but also in the proportion of water in the lower layer:

$$q_{12} = q_0 \left(1 + x_9 \left(\frac{S_2}{x_2} \right)^{x_{10}} \right) \left(\frac{S_1^F}{(1 - x_3)x_1} \right)$$

where q_0 is the base flow when the lower layer is completely filled with water.

Experimenting with structures $s_5 = 1$ and $s_5 = 2$ suggests that when x_3 is very high, the two structures give similar results. For moderate values of x_3, $s_5 = 2$ leads to smaller peaks and also more runoff some time after rainfall events, when both structures are run with the same x_8. But it is normally possible to choose two values for x_8, one for $s_5 = 1$ and one for $s_5 = 2$, such that the two structures give relatively little difference. If x_3 is very low, then the two structures vary significantly in unpredictable ways, and it was not possible to find two values of x_8 that could prevent these differences.

Finally, $s_5 = 3$ seems to give identical results to $s_5 = 1$ except when rainfall is high, when it gives rather more runoff immediately after the rainfall.

2.5 Evapotranspiration

There are two options available for evapotranspiration. In both cases, evapotranspiration from each tension zone is proportional to the proportion of the zone that contains water. For $s_6 = 2$ (`sequential`), potential evapotranspiration is taken from the upper layer first, and then from the lower layer only if there is any potential left:

$$\begin{aligned} e_1 &= E \frac{S_1^T}{x_3 x_1} \\ e_2 &= (E - e_1) \frac{S_2^T}{x_3 x_2}. \end{aligned}$$

If there is no tension zone in the upper layer, then S_1^T is taken to be the minimum of S_1 and $x_3 x_1$, and similarly for the lower layer.

For $s_6 = 1$ (`rootweighting`), potential evapotranspiration is split between the upper and lower layers according to the proportion of roots in the upper layer (x_6)

$$e_1 = E x_6 \frac{S_1^T}{x_3 x_1}$$
$$e_2 = E(1 - x_6) \frac{S_2^T}{x_3 x_2}.$$

If $s_2 = 3$, so there are two tension zones in upper layer, the evapotranspiration equations for the two tension zones are adapted from the above equations in the obvious way. If $s_3 = 3$ or 4, the lower layer is infinitely large so $e_2 = 0$. This explains why there is no evapotranspiration term in the differential equation for an unlimited lower layer.

When there is an unlimited lower layer ($s_3 = 3$ or 4), there is no evapotranspiration possible from the lower layer. Hence, there is always more evapotranspiration with $s_6 = 2$, and so discharge is always lower. It seems that in fact $s_6 = 2$ almost always gives lower discharge than $s_6 = 1$ even when the lower layer is limited, although when x_6 is near 1 there is occasionally less discharge with $s_6 = 1$. This suggests that evapotranspiration from the lower layer does not have much impact on discharge, because if x_6 is near 1, upper-layer evapotranspiration for $s_6 = 1$ is only a little less than for $s_6 = 2$, with lower-level evapotranspiration potentially being quite different. One might think the observed behaviour could be because potential evapotranspiration is never satisfied from the upper layer in the sequential system, but unrealistically high evapotranspiration data was tried as well, with the same results.

When x_6 is low, the differences in discharge between $s_6 = 1$ and $s_6 = 2$ can be quite high. It seems that the differences tend to occur when there is a small rainfall event. When the peak rainfall is high or a rainfall event is long, the differences are relatively minor. For $s_6 = 1$, it seems that discharge is uniformly higher the lower x_6 is.

2.6 Interflow

Interflow is a process by which a small amount of water leaves the upper layer from the free zone, to runoff. If $s_7 = 1$ (`intflwnone`), there is no interflow, so $q_{if} = 0$. If $s_7 = 2$ (`intflowsome`), interflow is proportional to the proportion of water in the free zone:

$$q_{if} = x_{12} \left(\frac{S_1^F}{(1 - x_3) x_2} \right).$$

Enabling interflow will typically increase discharge slightly, with the magnitude of the difference dependent upon x_{12}. Occasionally during a period of frequent rainfall, interflow can cause some of the discharge peaks to be a little lower, so interflow can't be used simply as a way of tuning the height of peaks. With model structures that admit shallow falling limbs, such as $s_3 = 1$ or $s_3 = 3$, interflow can cause these limbs to fall much more steeply. Indeed, when interflow is high, the choice of s_3 can be much less important.

For the occasional model structure, enabling interflow can cause extremely wild and unpredictable behaviour. This seems like an error. The specific causes of this rare problem have yet to be determined.

2.7 Time delay

The model allows a delay to be imposed for surface runoff to lead to discharge, so that the discharge generated at a given timestep is divided between some following timesteps. The $s_8 = 2$ (`rout_gamma`) models the time delay using a gamma distribution with mean time delay x_{22}. Routing can instead be turned off using $s_8 = 1$ (`no_routing`). In practice it seems that imposing a time delay is necessary to get sensible results, although for some catchments that time delay does not have to be very large.

Parameter	RHydro	Description	Clark	Structure
x_1	`maxwatr_1`	Maximum storage in the upper layer	$S_{1,\max}$	All
x_2	`maxwatr_2`	Maximum storage in the lower layer	$S_{2,\max}$	All
x_3	`fracten`	Fraction of total storage that is tension storage	ϕ_{tens}	All
x_4	`frchzne`	Fraction of tension storage that is in the primary tension zone (upper layer)	ϕ_{rchr}	$s_2 = 3$
x_5	`fprimqb`	Fraction of free storage that is in the primary free zone (lower layer)	ϕ_{base}	$s_3 = 2$
x_6	`rtfrac1`	Fraction of roots in the upper layer	r_1	$s_6 = 1$
x_7	`percrte`	Percolation rate	k_u	$s_5 = 1$ or 2
x_8	`percexp`	Percolation exponent	c	$s_5 = 1$ or 2
x_9	`sacpmlt`	Percolation multiplier	α	$s_5 = 3$
x_{10}	`sacpexp`	Percolation exponent	ψ	$s_5 = 3$
x_{11}	`percfrac`	Fraction of percolation that goes to tension layer	κ	$s_3 = 2$
x_{12}	`iflwrte`	Interflow rate	k_i	$s_7 = 2$
x_{13}	`baserte`	Baseflow rate	k_s	$s_3 = 1$ or 4, or $s_4 = 3$
x_{14}	`qb_powr`	Baseflow exponent	n	$s_3 = 1$ or 4, or $s_4 = 3$
x_{15}	`qb_prms`	Baseflow rate	v	$s_3 = 3$
x_{16}	`qbrate_2a`	Baseflow rate for one free compartment	v_A	$s_3 = 2$
x_{17}	`qbrate_2b`	Baseflow rate for the other free compartment	v_B	$s_3 = 2$
x_{18}	`sareamax`	Maximum area that can be saturated (as a fraction)	$A_{i,\max}$	$s_4 = 2$
x_{19}	`axv_bexp`	Exponent for surface runoff	b	$s_4 = 1$
x_{20}	`loglamb`	Mean parameter for the topographic index distribution	λ	$s_4 = 3$ or $s_3 = 4$
x_{21}	`tishape`	Shape parameter for the topographic index distribution	χ	$s_4 = 3$ or $s_3 = 4$
x_{22}	`timedelay`	Mean time delay	μ_τ	$s_8 = 2$

Table 1: List of adjustable parameters, with description and model structure choices that activate them.

Switch	Description	Value	`RHydro`	x_i Activated
s_2	Upper layer structure	1	`onestate_1`	
		2	`tension1_1`	
		3	`tension2_1`	x_4
s_3	Lower layer structure	1	`fixedsiz_2`	x_{13}, x_{14}
		2	`tens2pll_2`	$x_5, x_{11}, x_{16}, x_{17}$
		3	`unlimfrc_2`	x_{15}
		4	`unlimpow_2`	$x_{13}, x_{14}, x_{20}, x_{21}$
s_4	Runoff	1	`arno_x_vic`	x_{19}
		2	`prms_varnt`	x_{18}
		3	`tmdl_param`	$x_{13}, x_{14}, x_{20}, x_{21}$
s_5	Percolation	1	`perc_f2sat`	x_7, x_8
		2	`perc_w2sat`	x_7, x_8
		3	`perc_lower`	x_9, x_{10}
s_6	Evaporation	1	`rootweighting`	x_6
		2	`sequential`	
s_7	Interflow	1	`intflwnone`	
		2	`intflwsome`	x_{12}
s_8	Time Delay	1	`no_routing`	
		2	`rout_gamma`	x_{22}

Table 2: Model structure switches and possible values, and the parameters they activate.