MUCM Presentation

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Who am I?

- Jonathan Cumming
- Research Associate at Durham University
- Working on WP1.3: *Multiscale Analysis* with Michael Goldstein
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- Research Associate at Durham University
- Working on WP1.3: *Multiscale Analysis* with Michael Goldstein
- PhD in Statistics (2006) - *Clinical Decision Support*
  - Longitudinal data analysis
  - Variable selection
  - Visualisations
All computer models have an intrinsic level of complexity
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More complex models are typically slower and more expensive, but also more accurate
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Differences in model complexity can be attributable to:

- Finer (coarser) grid resolutions
- More (less) accurate solver
- More (less) complex underlying mathematics
Multiscale Models

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■ Differences in model complexity can be attributable to:
  ◆ Finer (coarser) grid resolutions
  ◆ More (less) accurate solver
  ◆ More (less) complex underlying mathematics

■ These differences in complexity can result in sets of different but closely related models

■ Results from models of one level of complexity will be related to those of a higher (lower) level
Linked Approach

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- We have simulators which are linked, so create linked emulators for these models
- How can we do this
  - Link via regression coefficients of emulators for the models
  - Link via emulating the difference between the fast and the slow models
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Used linked emulators to exploit cheapness of low-level models to inform us about high-level model

Emulators over small regions of input space require fewer model runs to reproduce output accurately
Goals

- Develop linked emulation methodology
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- Incorporate and exploit spatio-temporal features
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- Prediction
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- What resolution do we need to attain specified levels of accuracy?
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Related problems:

- Design for multiscale models
- Calibration of multiscale models
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So far...

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1. Familiarisation with existing methodology in the literature
2. Development of a ‘simple’ computer model
3. Exploration and implementation of emulation methods
The Daisyworld Model

The Daisyworld Model

- The original Daisyworld is composed of:
  - A zero-dimensional planet of unit area illuminated by a sun
  - Two distinct species of vegetation covering the planet’s surface - black daisies and white daisies
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- The key feature of the model is the different colour of daisy - different colours mean different *albedos*

- White daisies reflect more heat and so are cooler; black daisies absorb more heat and so are hotter

- This creates local temperature variation which affects the temperature of Daisyworld as a whole
The Daisyworld Model 2

- Inputs:
  - Initial size of daisy populations
  - Daisy albedos, bare earth albedo
  - Insolation - ‘sun strength’
  - Other parameters: death rate, optimal growth temperature, heat absorption factor, etc.
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- **Outputs:**
  - Planetary temperature
  - Daisy population size
  - Can be:
    - Time series, e.g. if forcing the model
    - Single values, e.g. equilibrium state
The Original Experiment

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So far...
The Daisyworld Model
The Daisyworld Model 2

The Original Experiment

Daisyworld+
Daisyworld+ 2
Some early results
Multiscale Emulation
Multiscale Emulation

What’s next?
Daisyworld can be ‘easily’ extended to a more interesting model:

- Multiple types of daisies:
Daisyworld can be ‘easily’ extended to a more interesting model:

- **Multiple types of daisies:**
  - Introduces a multiscale dimension to the model
  - No longer just 2 daisies
  - Have ‘families’ of black and white daisy species of varying colour
  - Now a multiscale model
Daisyworld+ 2

- Introduce extra trophic levels:
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- Rabbits can control daisy populations allowing for a more interesting equilibrium state
- Foxes can control rabbit populations
- Both cause changes in the model’s output
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- Model vs. reality

Available for all to enjoy!
Some early results

The model:

- 2 daisy model - daisy albedos are inputs, equilibrium planetary temperature is output
- 2 levels of complexity - coarse solver and fine solver
- Use Kennedy and O’Hagan (2000) framework:

\[ z_1(x) = \delta_1(x)z_2(x) = \rho_1 z_1(x) + \delta_2(x) \]

where (conditional on hyperparameters) \( \delta_t(x) \) is a stationary GP with mean \( h(\cdot)^T \beta_t \) and covariance function \( c_t(x, x') = \text{Cov}[\delta_t(x), \delta_t(x')] \).

- Include linear terms and linear interactions in mean function
- Captures majority of variation \( (R^2 \approx 0.9) \)
Multiscale Emulation

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What’s next?
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- Explore different methods
- Look more carefully at the performance of the various methods
- Try out different models
- Develop a simplified linked emulation framework for multiscale simulators